Bi-Gaussianized Calibration Revisited: Reducing the potential for overstating strength of evidence

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Disclaimer

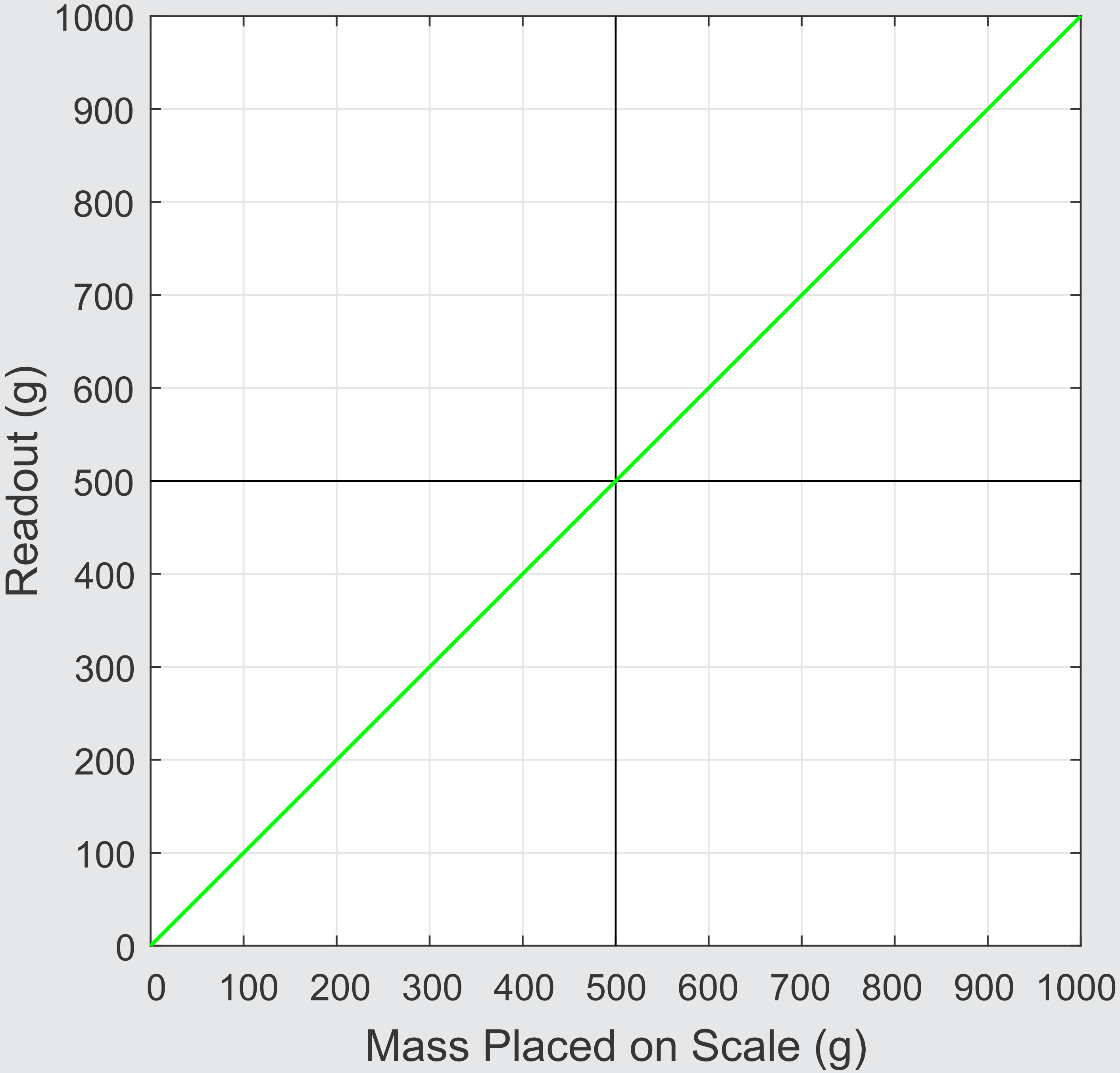
- All opinions expressed are those of the presenter and, unless explicitly stated otherwise, do not necessarily represent the opinions of the other authors nor the policies or positions of any organizations with which the authors are associated.

Slides, papers, and software

- Morrison G.S. (2024). **Bi-Gaussianized calibration of likelihood ratios**. *Law, Probability & Risk*, 23, mgae004. https://doi.org/10.1093/lpr/mgae004
- Morrison G.S., Vegeer P., Borchert D. (2026). **Bi-Gaussianized calibration revisited: Reducing the potential for overstating strength of evidence**. Manuscript in preparation.

https://forensic-data-science.net/calibration-and-validation/#biGauss

Calibrated scales

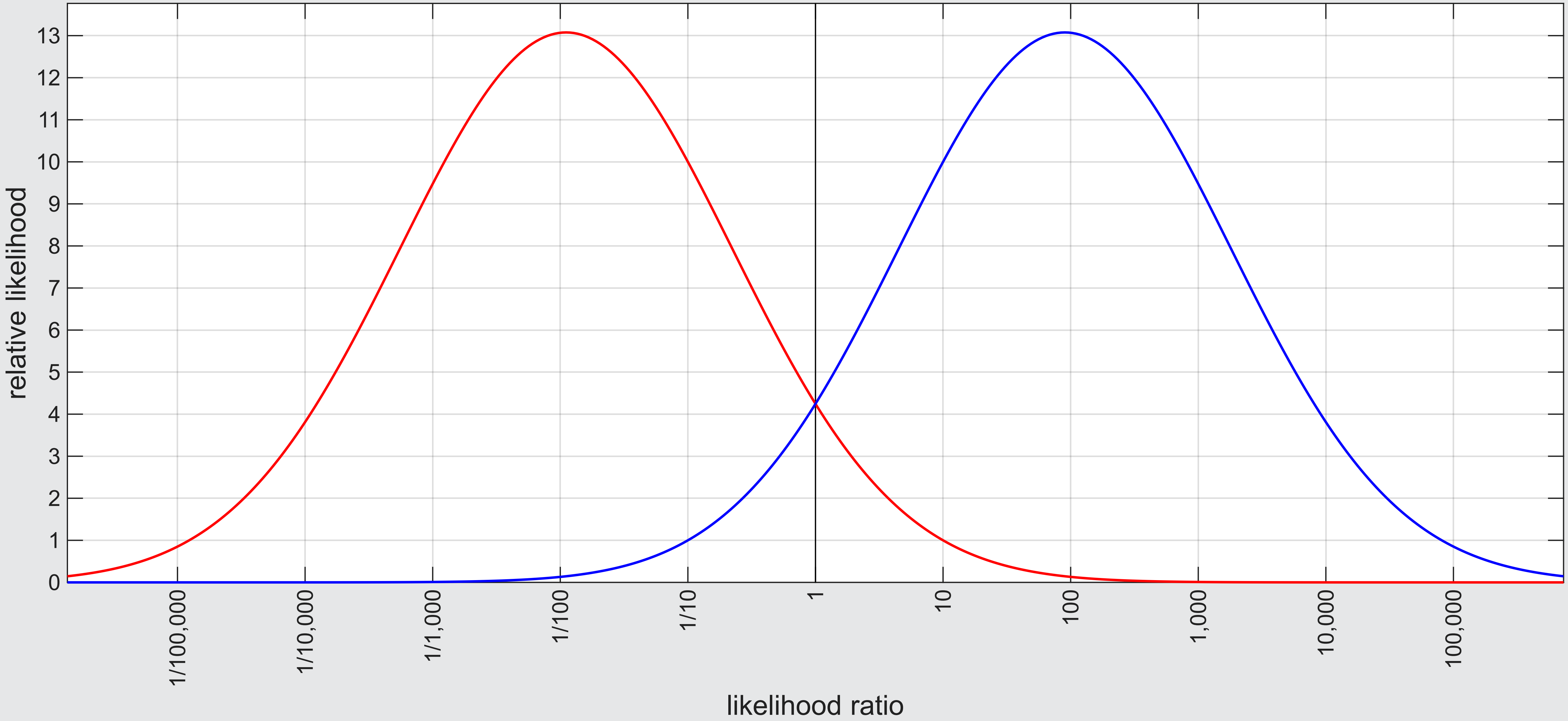


Calibrated likelihood ratios

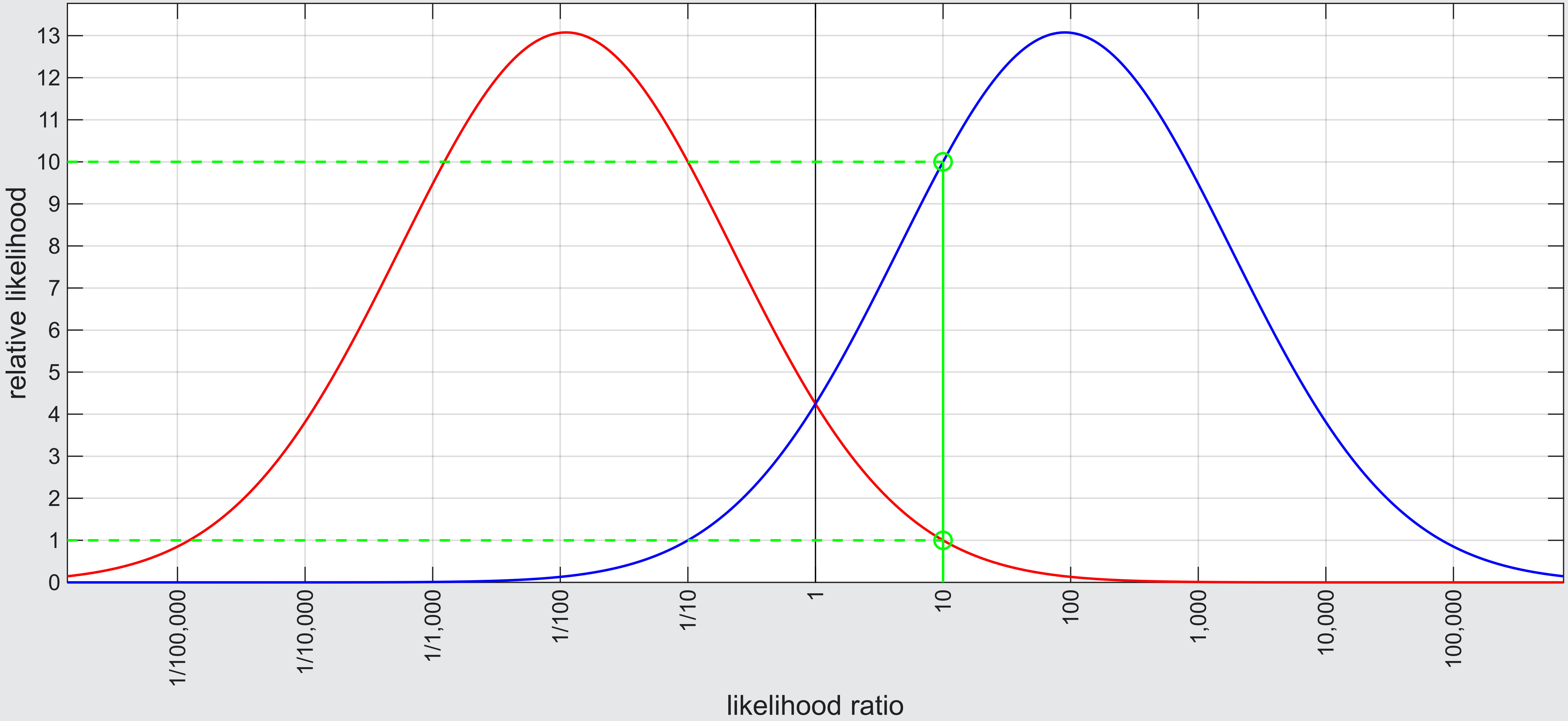
The likelihood ratio of the likelihood ratio is the likelihood ratio.

$$\Lambda = \frac{f(\Lambda \mid H_{\mathrm{s}})}{f(\Lambda \mid H_{\mathrm{d}})}$$

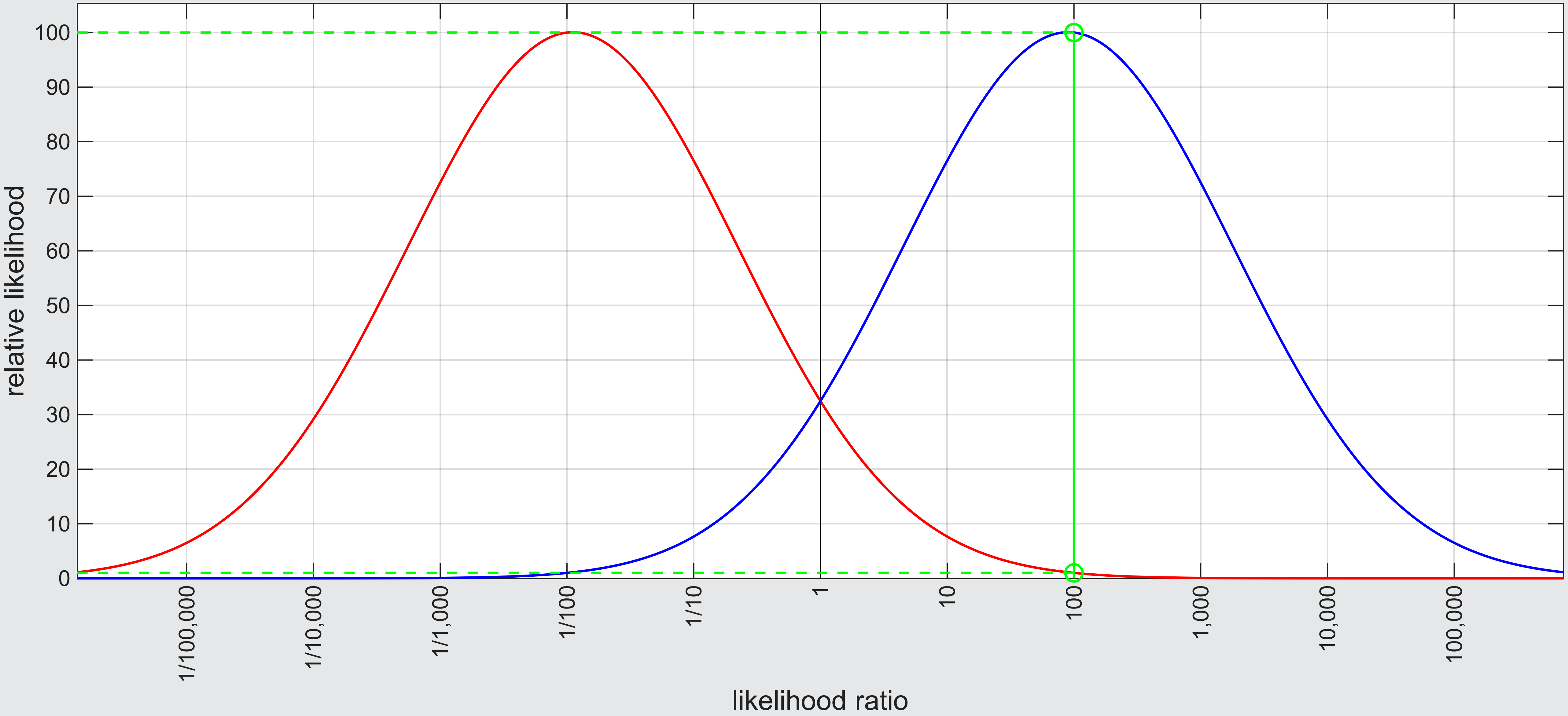
Calibrated likelihood ratios



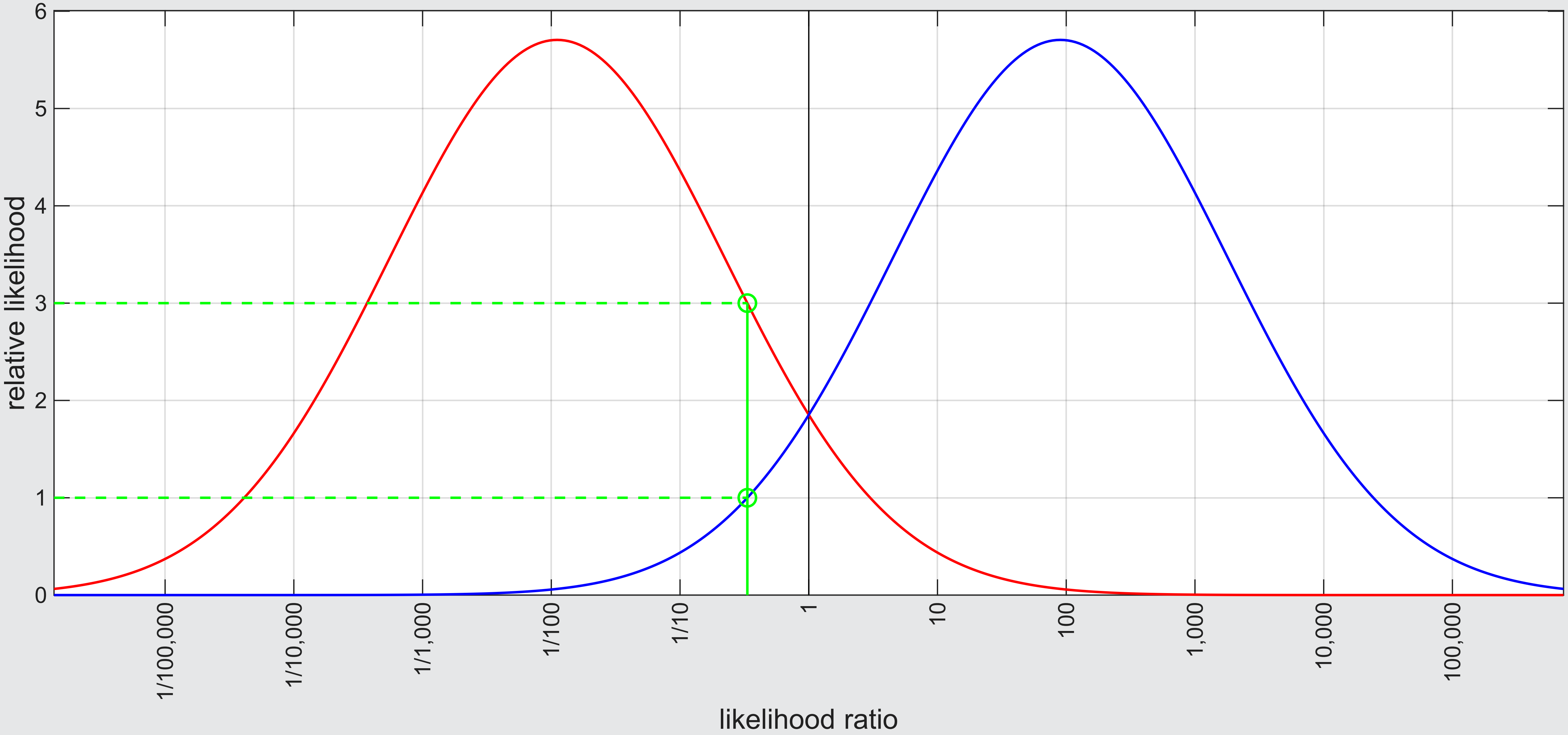
Calibrated likelihood ratios



Calibrated likelihood ratios

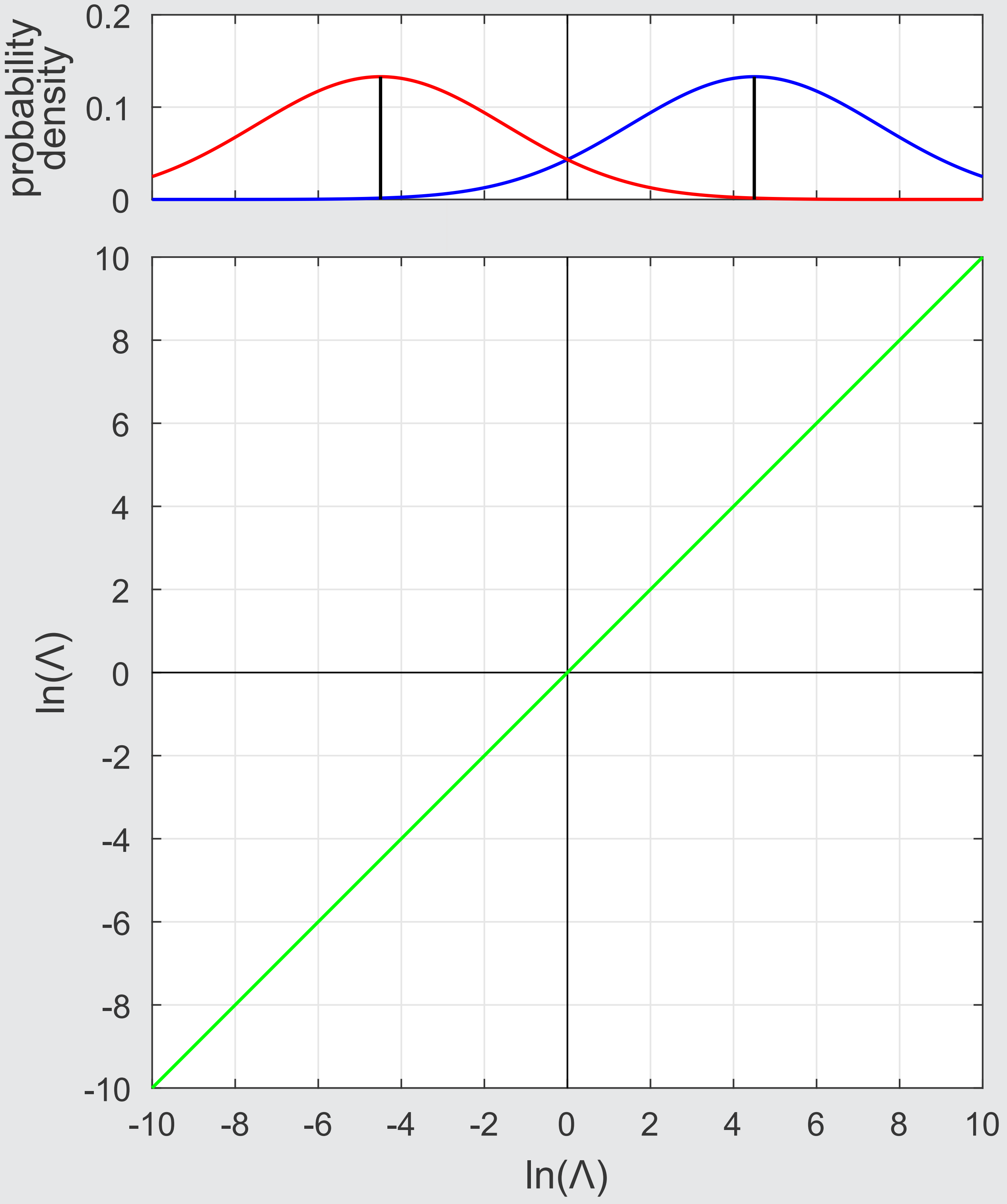


Calibrated likelihood ratios



Calibrated likelihood ratios

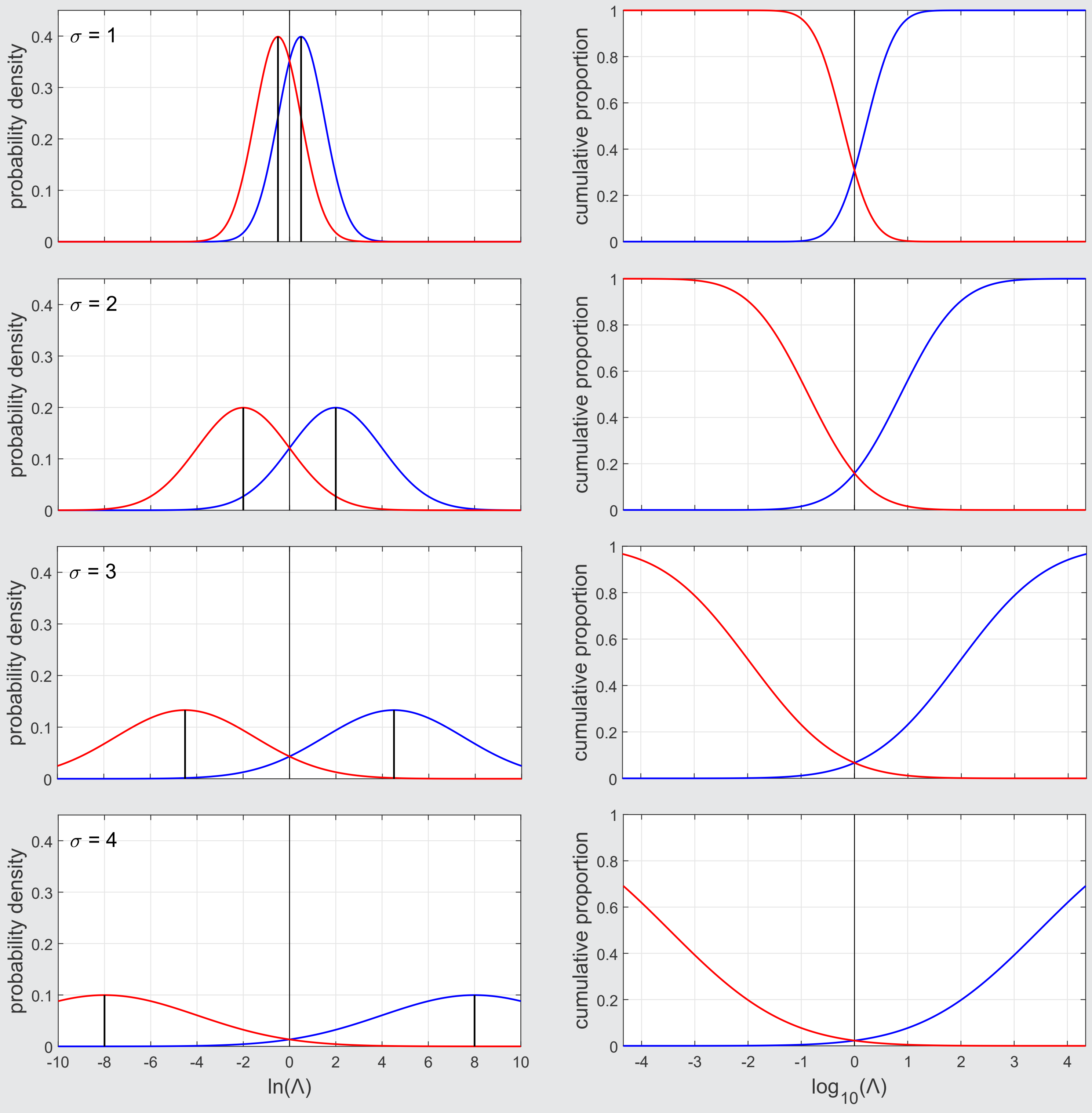
$$\ln(\Lambda) = \frac{f(\ \ln(\Lambda) \mid H_{\mathrm{s}}\ )}{f(\ \ln(\Lambda) \mid H_{\mathrm{d}}\ )}$$



Calibrated likelihood ratios

- Perfectly calibrated bi-Gaussian systems

- Same-source and different-source distributions:
    - $\ln(\Lambda)$ scale
    - Gaussian
    - same variance
    - $\mu_s = +\dfrac{\sigma^2}{2}$
    - $\mu_d = -\dfrac{\sigma^2}{2}$



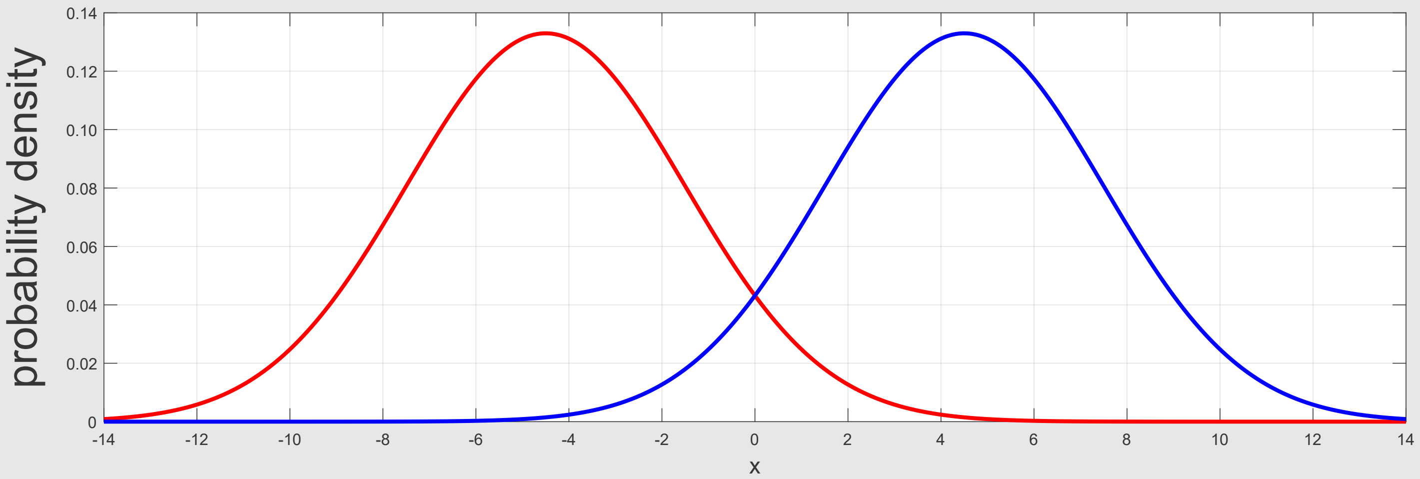
Bi-Gaussianized calibration

- Monotonically (but non-linearly) map the distribution of uncalibrated $\log(\Lambda)$ toward the Gaussian-mixture distribution for $\ln(\Lambda)$ of a perfectly calibrated bi-Gaussian system.

Bi-Gaussianized calibration

- Perfectly calibrated bi-Gaussian system

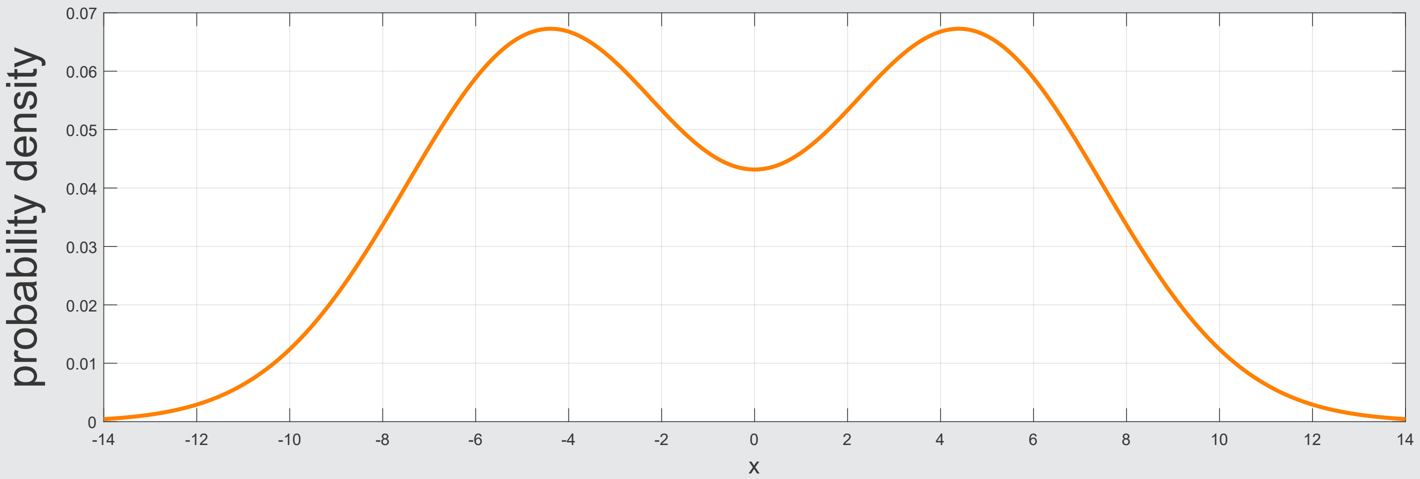
- Same source and different source
  - probability density



Bi-Gaussianized calibration

- Perfectly calibrated bi-Gaussian system

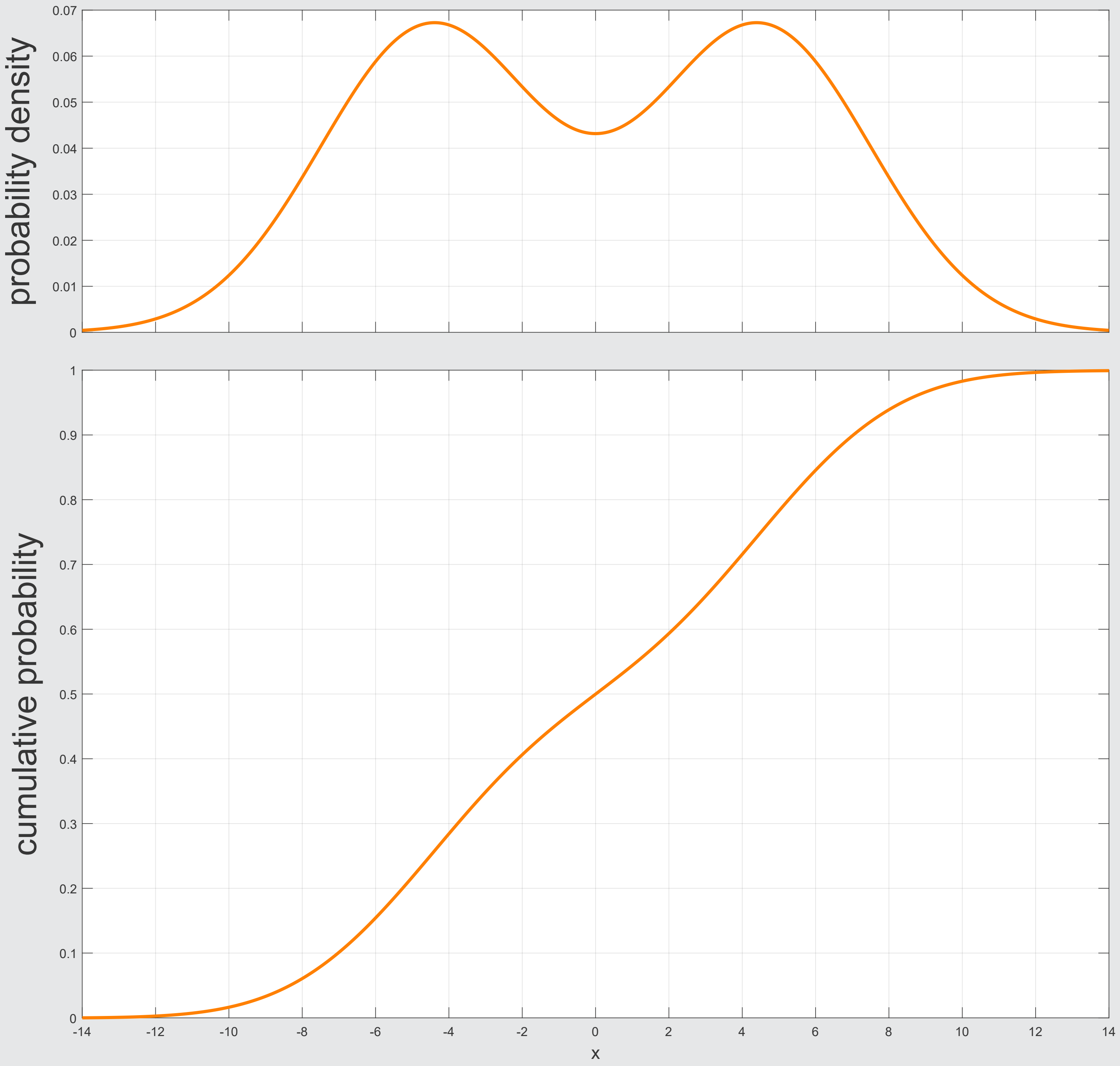
- Gaussian mixture model
  - probability density



Bi-Gaussianized calibration

- Perfectly calibrated bi-Gaussian system

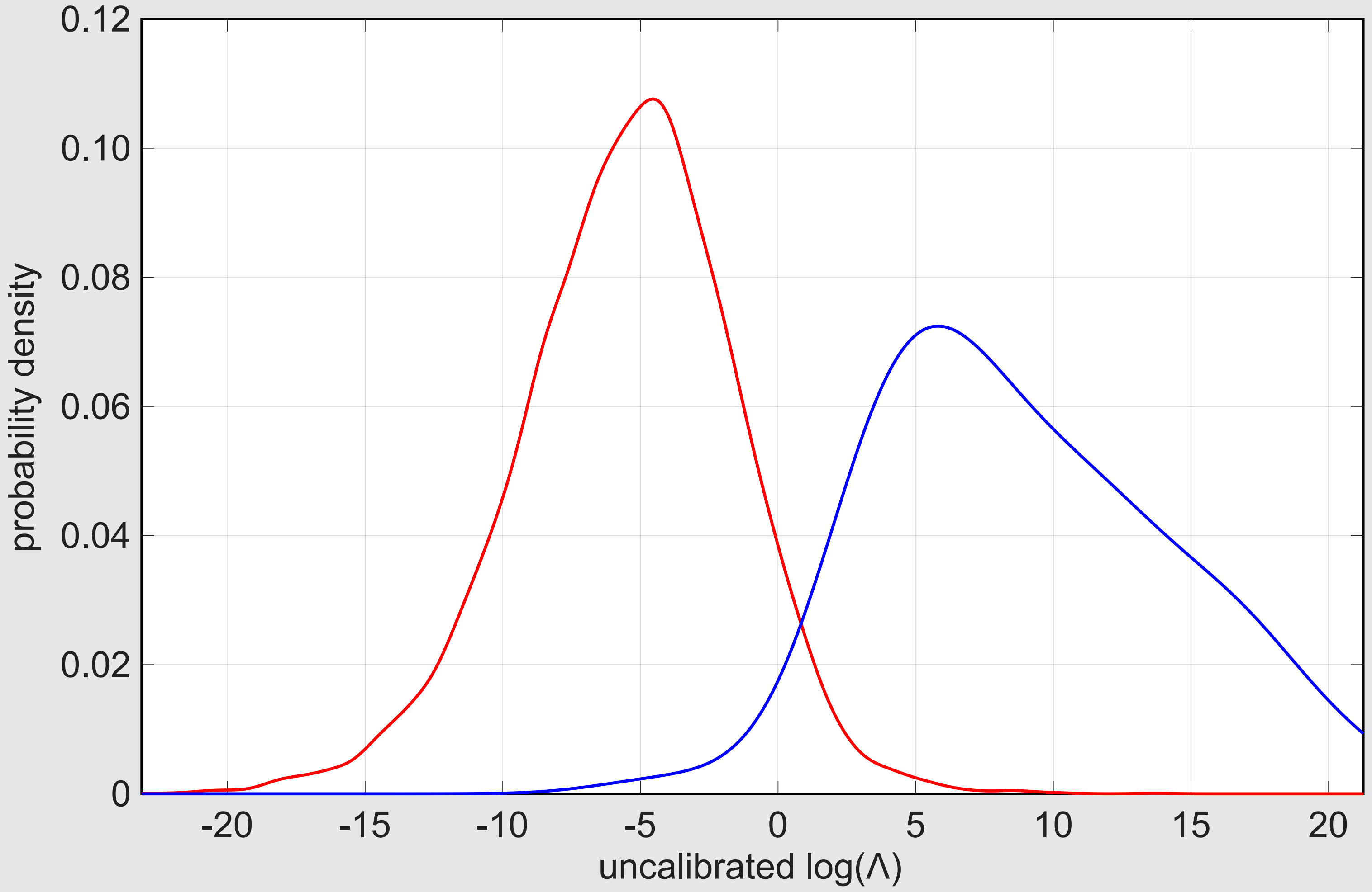
- Gaussian mixture model
  - probability density
  - cumulative probability



Bi-Gaussianized calibration

- Forensic-voice-comparison system
  - Uncalibrated log(Λ)

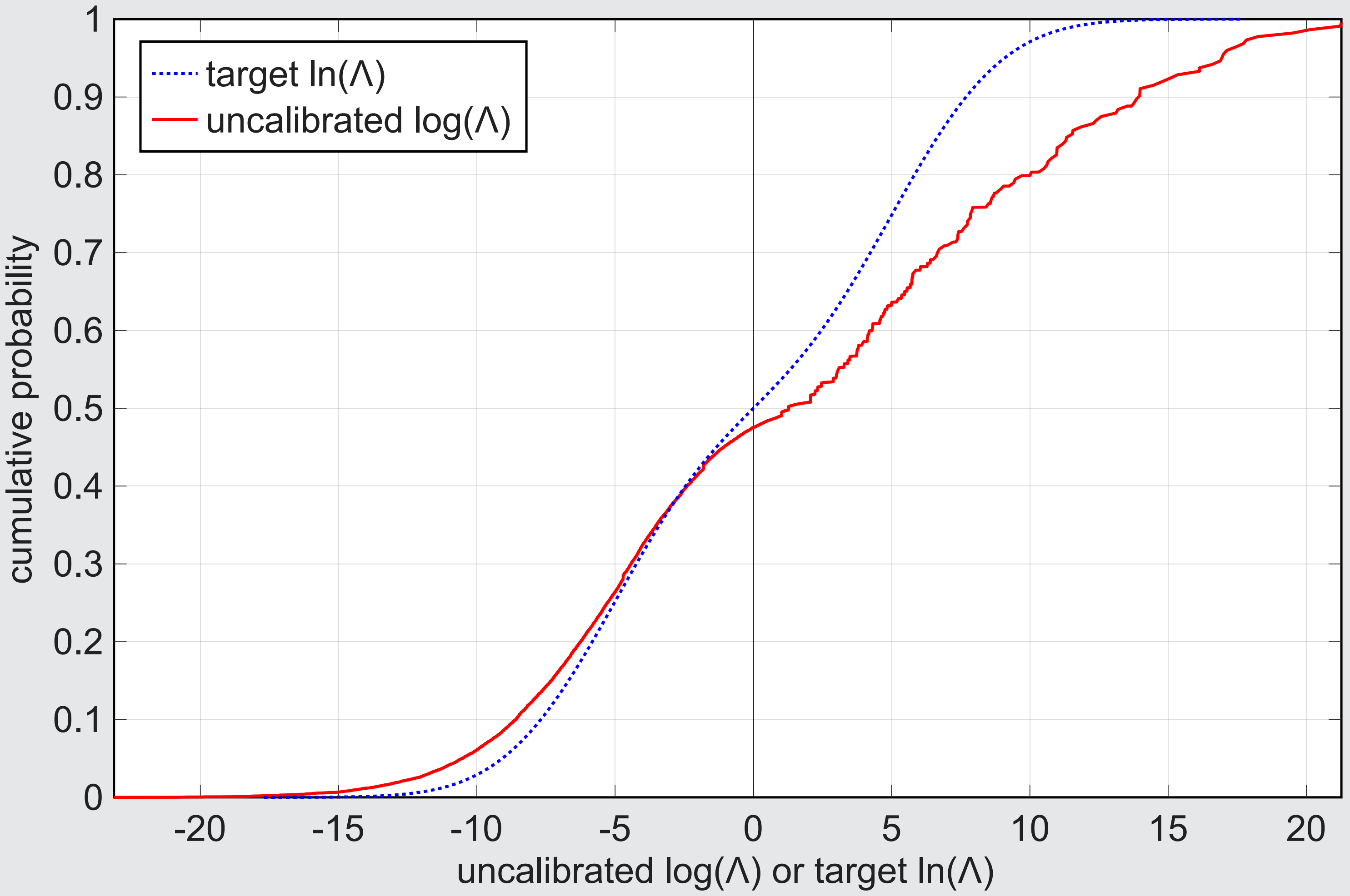
- Same source and different source
  - probability density



Bi-Gaussianized calibration

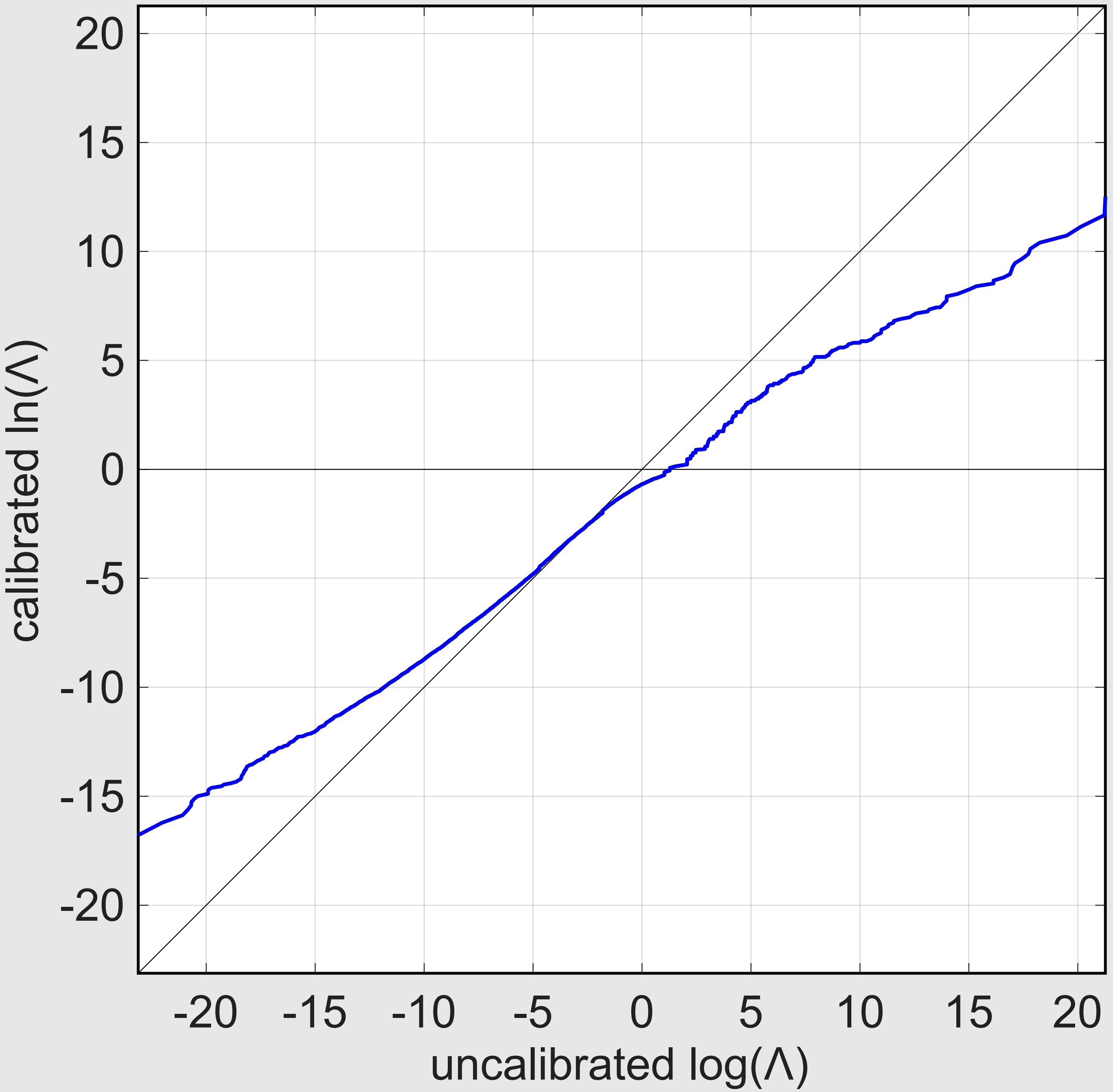
- Forensic-voice-comparison system
  - Uncalibrated log(Λ)
  - empirical cumulative probability
  - same weight to same-source & different-source categories

- Perfectly calibrated bi-Gaussian system
  - Gaussian-mixture model
  - cumulative probability



Bi-Gaussianized calibration

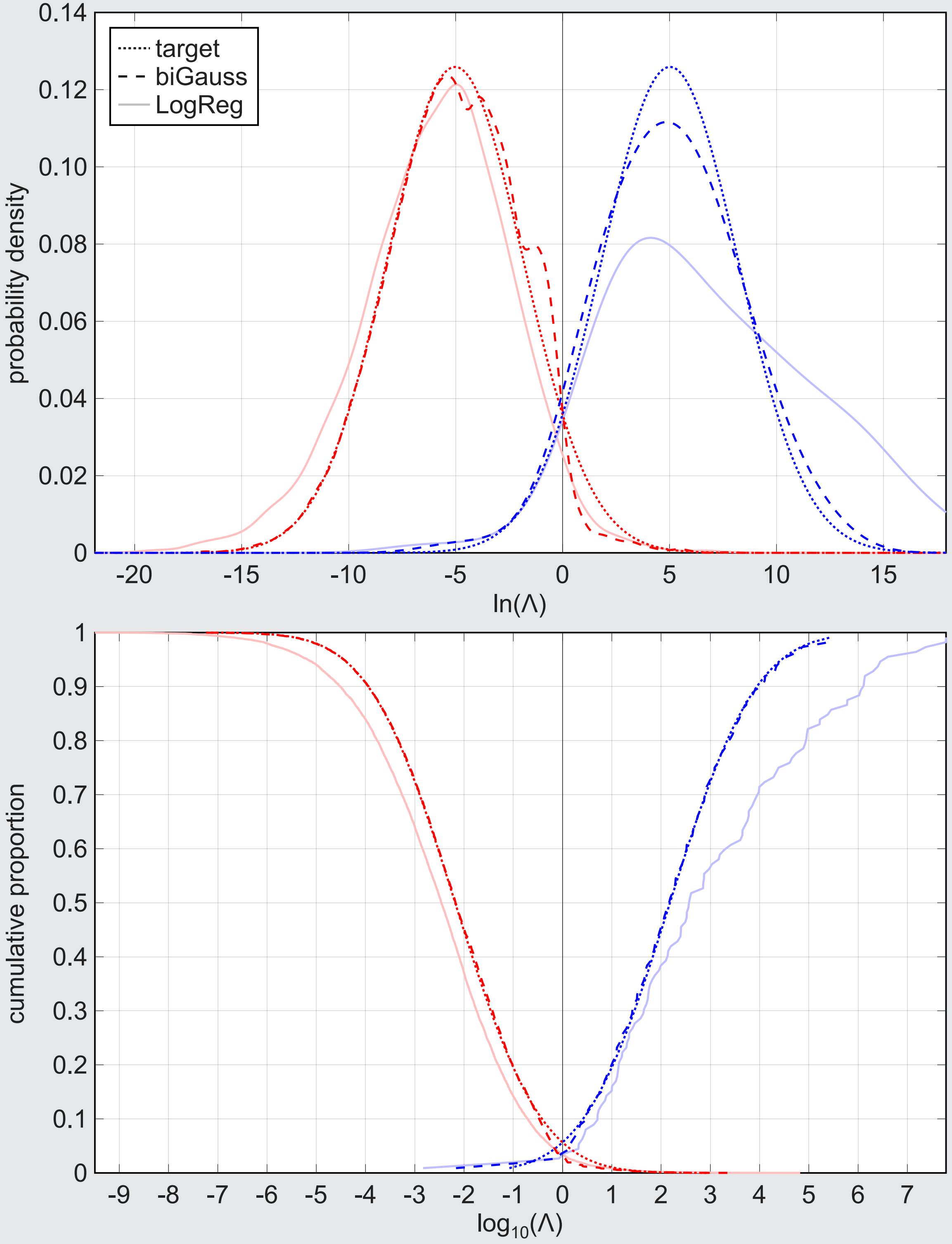
- Forensic-voice-comparison system
  - Uncalibrated log(Λ) to calibrated ln(Λ) mapping function



Bi-Gaussianized calibration

- Forensic-voice-comparison system

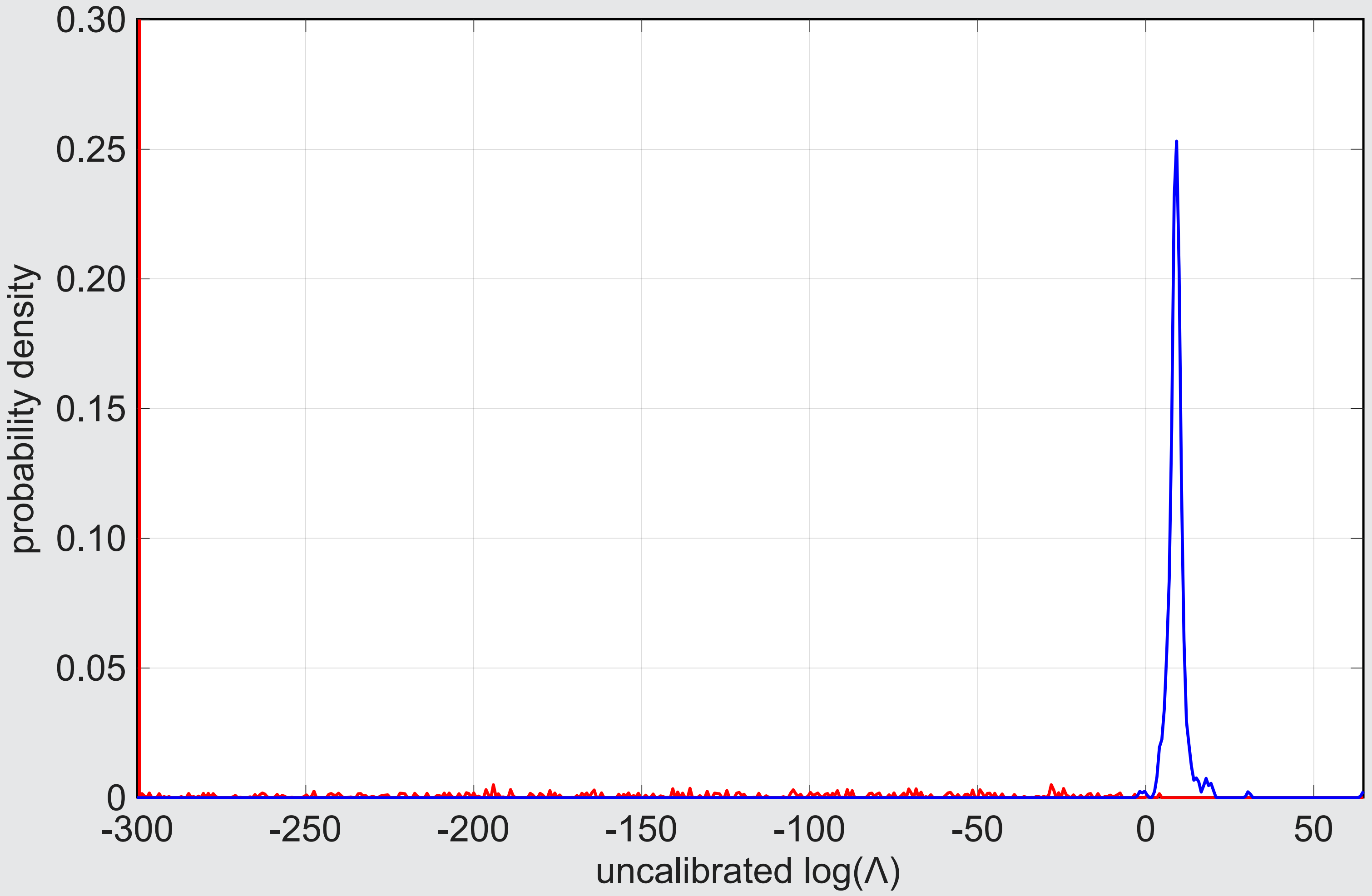
- Calibrated ln(Λ)
  - target perfectly calibrated system
  - bi-Gaussianized calibration
  - logistic-regression calibration



Bi-Gaussianized calibration

- Glass fragments
    - Uncalibrated log(Λ)

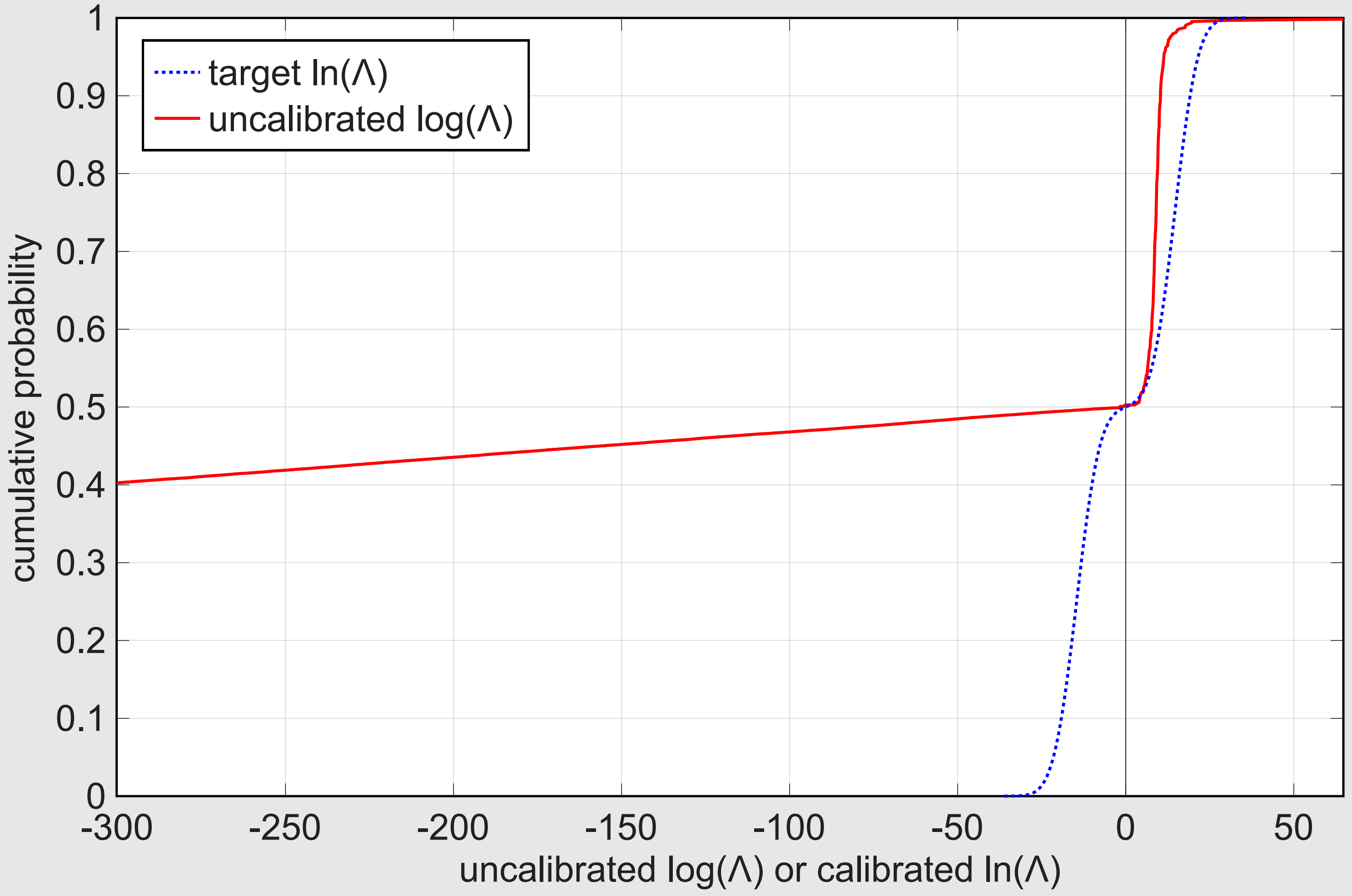
- Same source and different source
    - probability density



Bi-Gaussianized calibration

- Glass fragments
  - Uncalibrated log(Λ)
  - empirical cumulative probability
  - same weight to same-source & different-source categories

- Perfectly calibrated bi-Gaussian system
  - Gaussian-mixture model
  - cumulative probability



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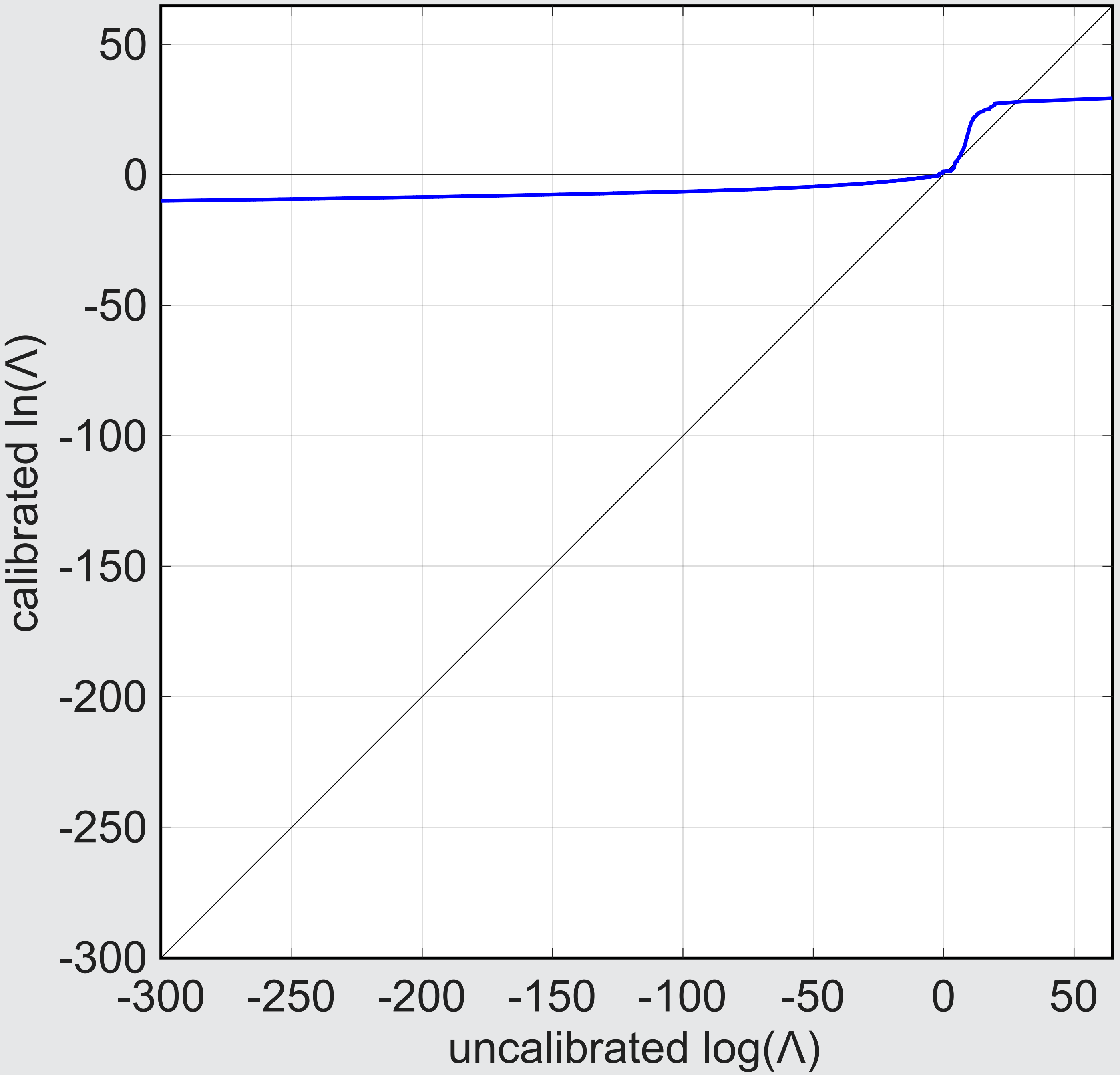

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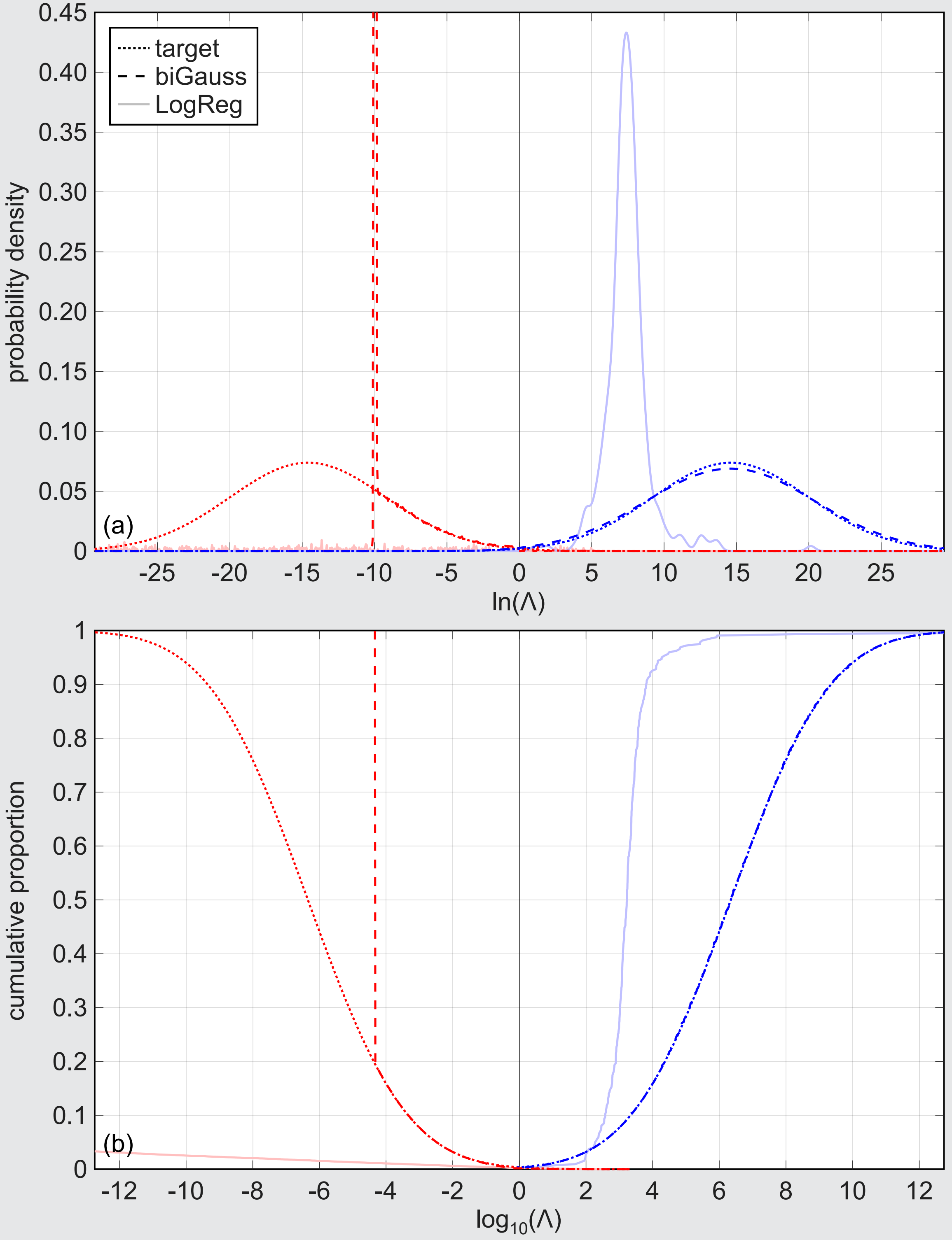

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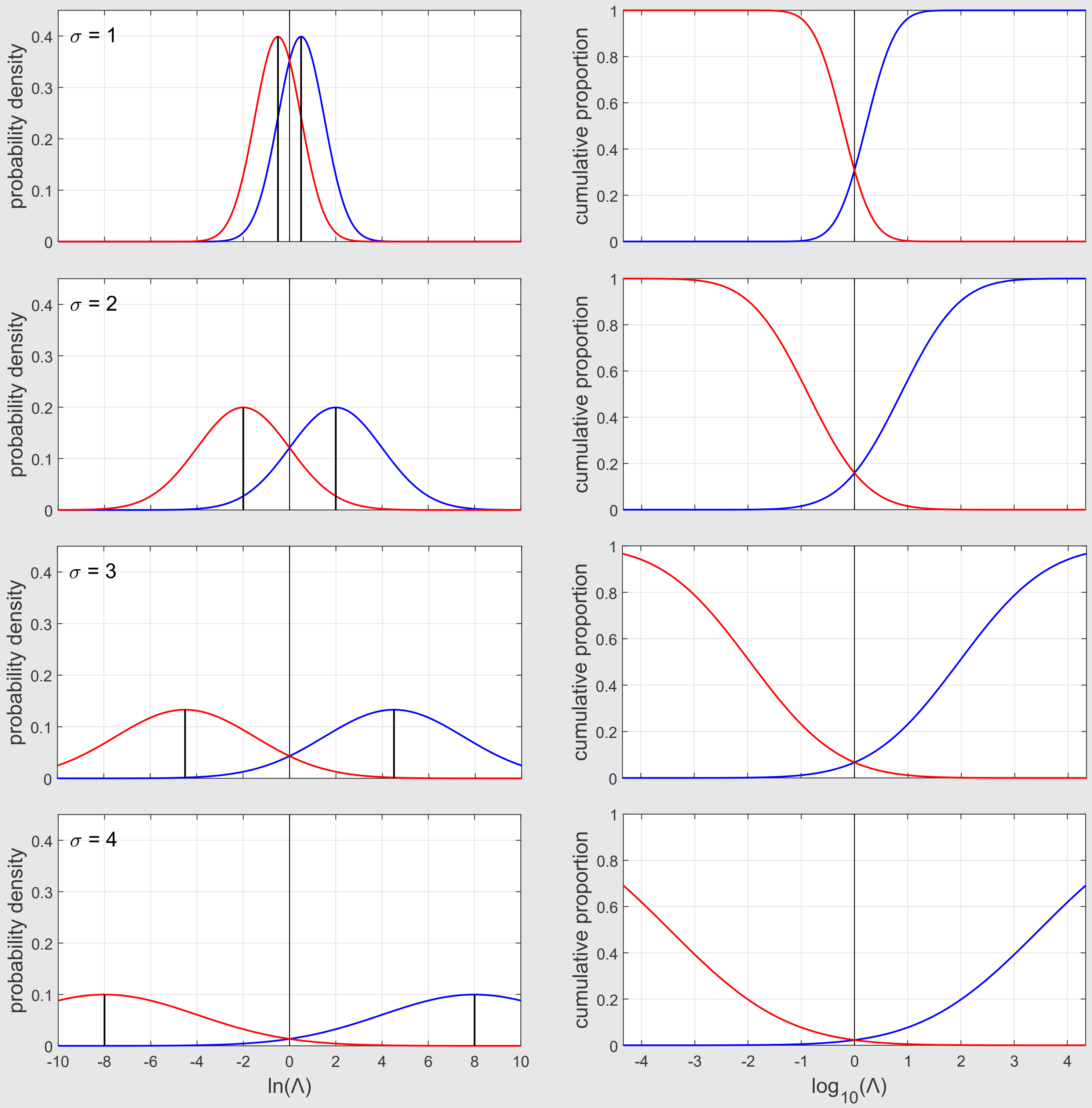
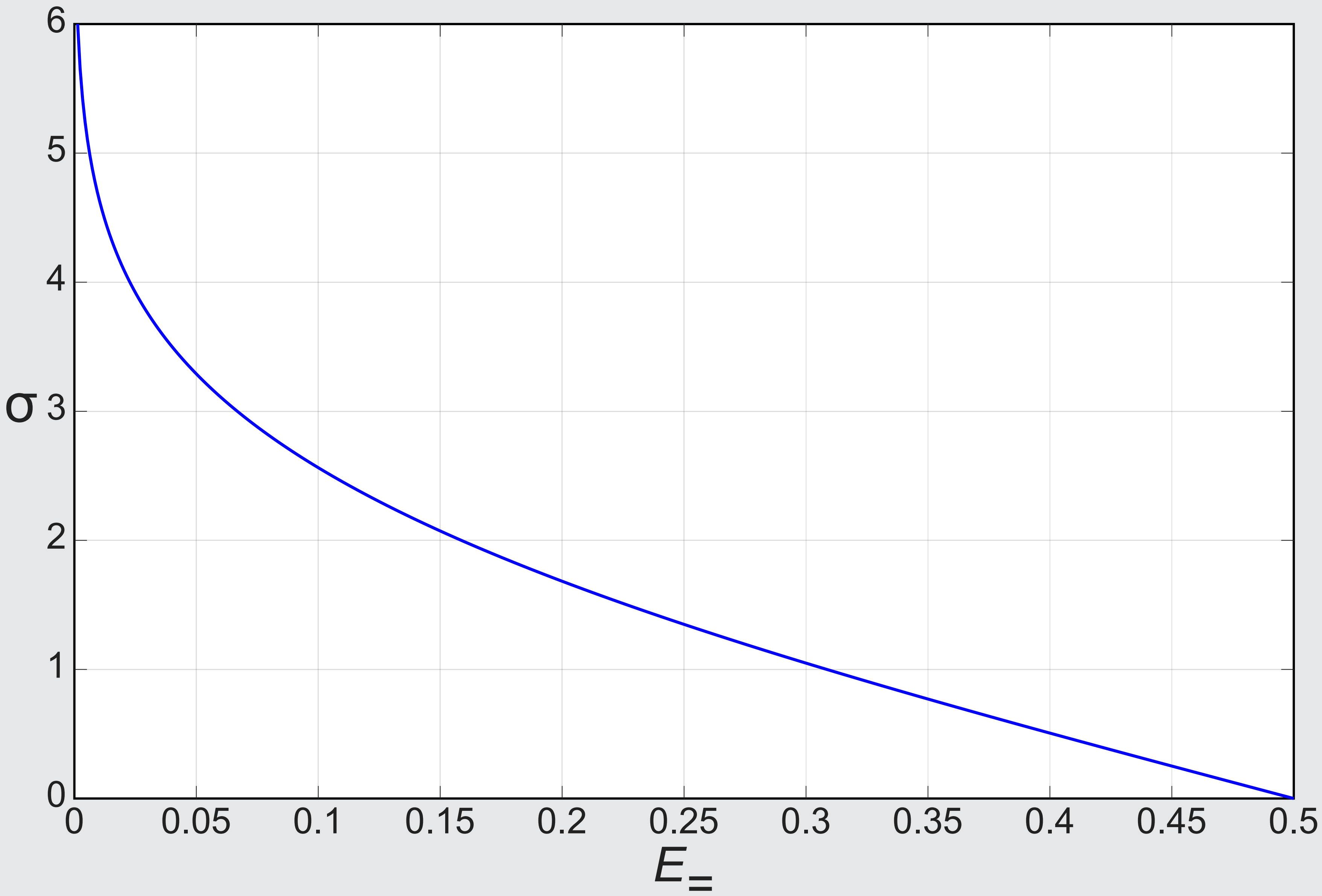


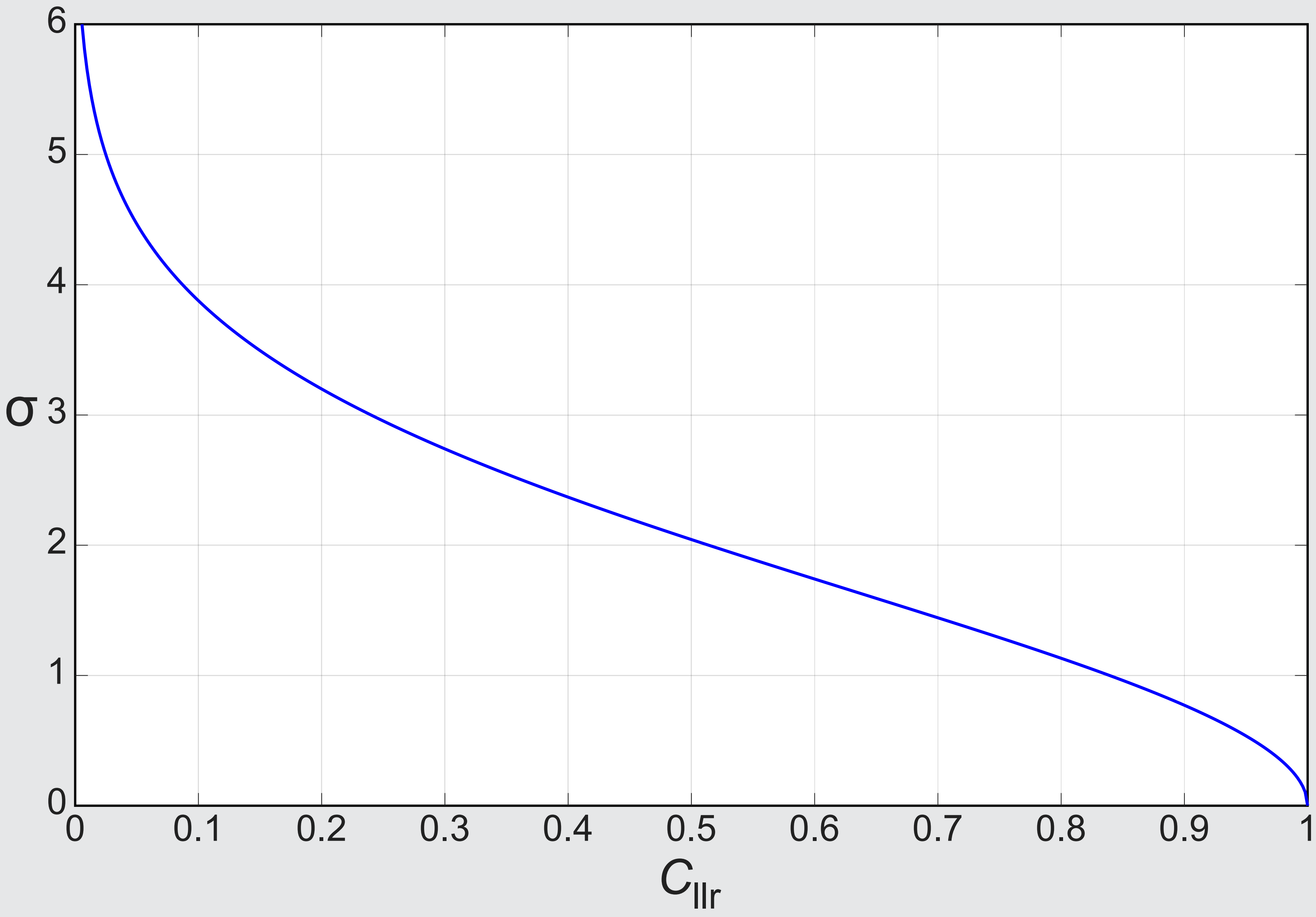
Bi-Gaussianized calibration

- Equal-error-rate to standard-deviation mapping function



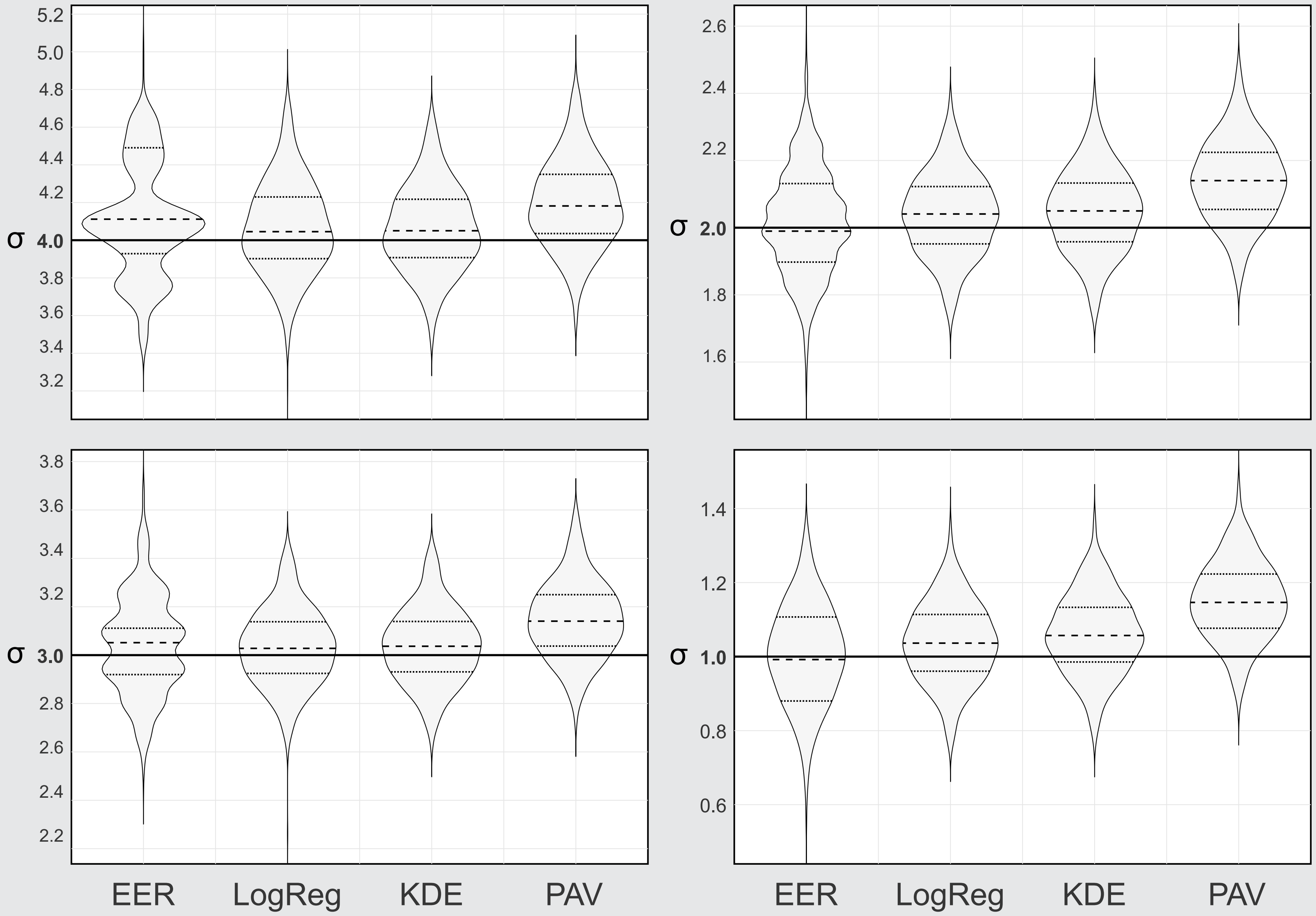
Bi-Gaussianized calibration

- Log-likelihood-ratio-cost to standard-deviation mapping function



Bi-Gaussianized calibration

- 2024 paper
- Monte Carlo simulation
  - σ ∈ {1,2,3,4}
  - 1000 sample sets per σ
  - 100 sources per set
- Methods
  - EER
  - LogReg
  - KDE
  - PAV



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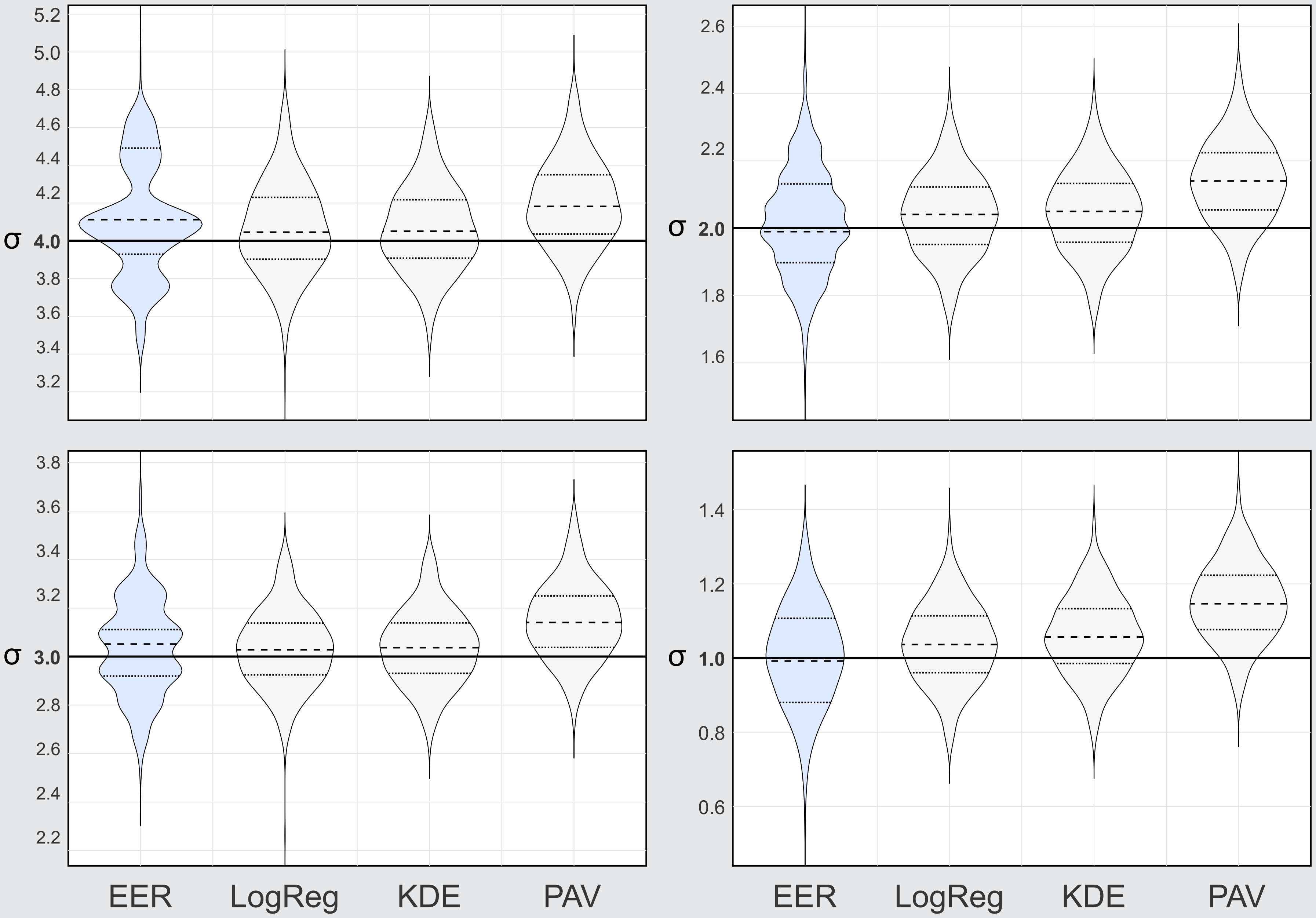

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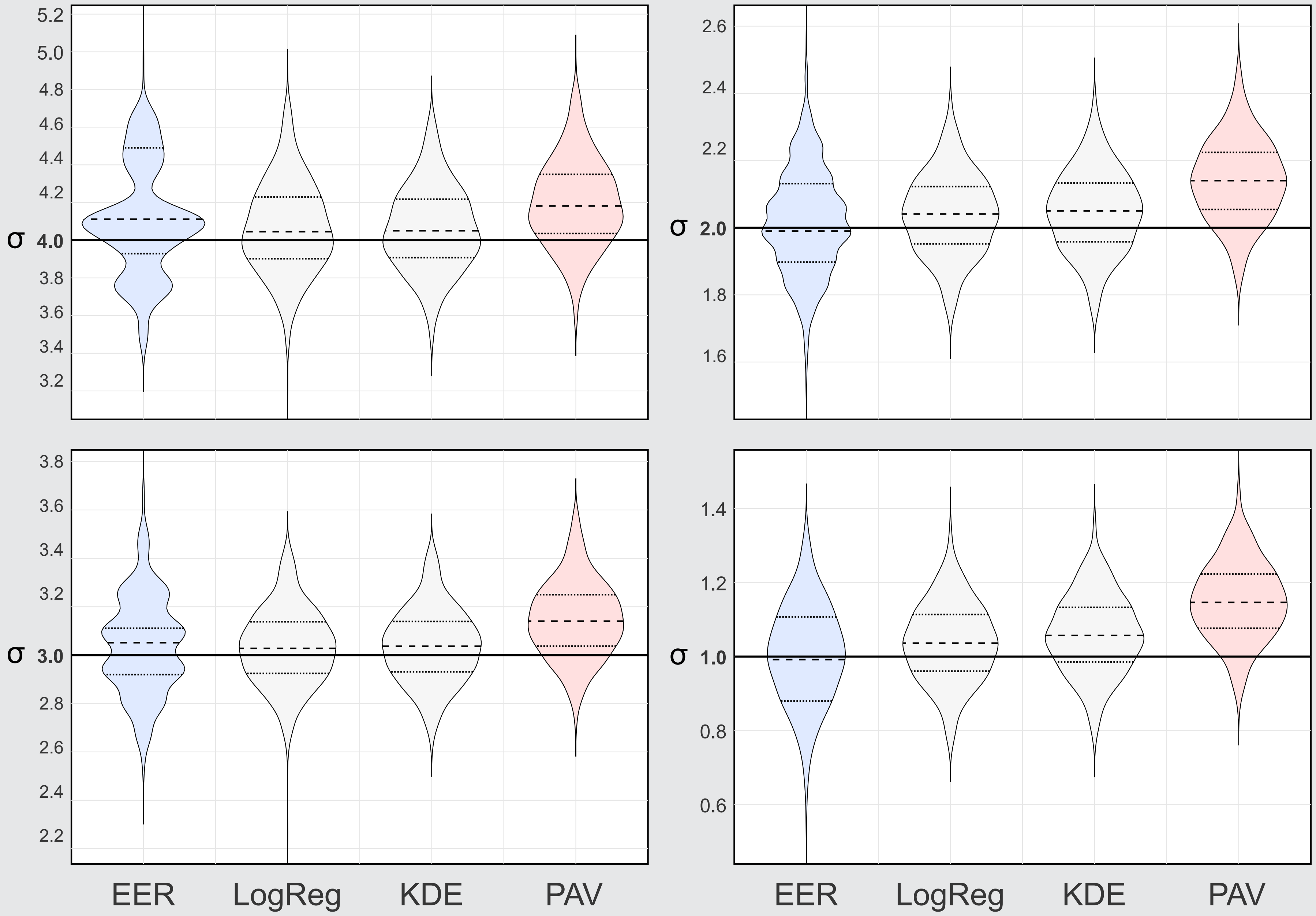
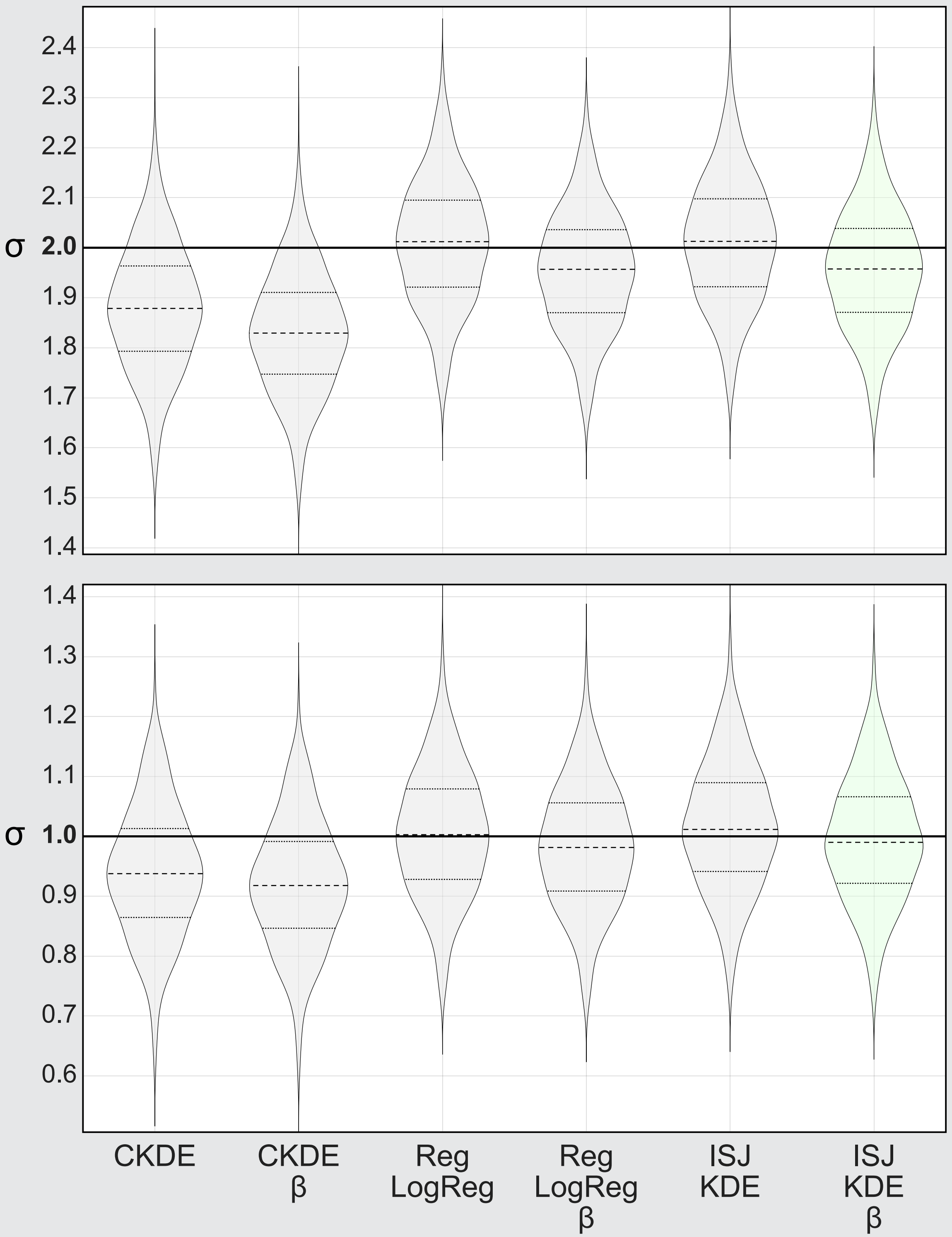


Bi-Gaussianized calibration

- 2026 paper
- Monte Carlo simulation
  - σ ∈ {1,2,3,4,5,6}
  - 1000 sample sets per σ
  - 100 sources per set
- Methods
  - CKDE
  - CKDE-β
  - RegLogReg
  - RegLogReg-β
  - ISJ-KDE
  - ISJ-KDE-β



Bi-Gaussianized calibration

- 2026 paper
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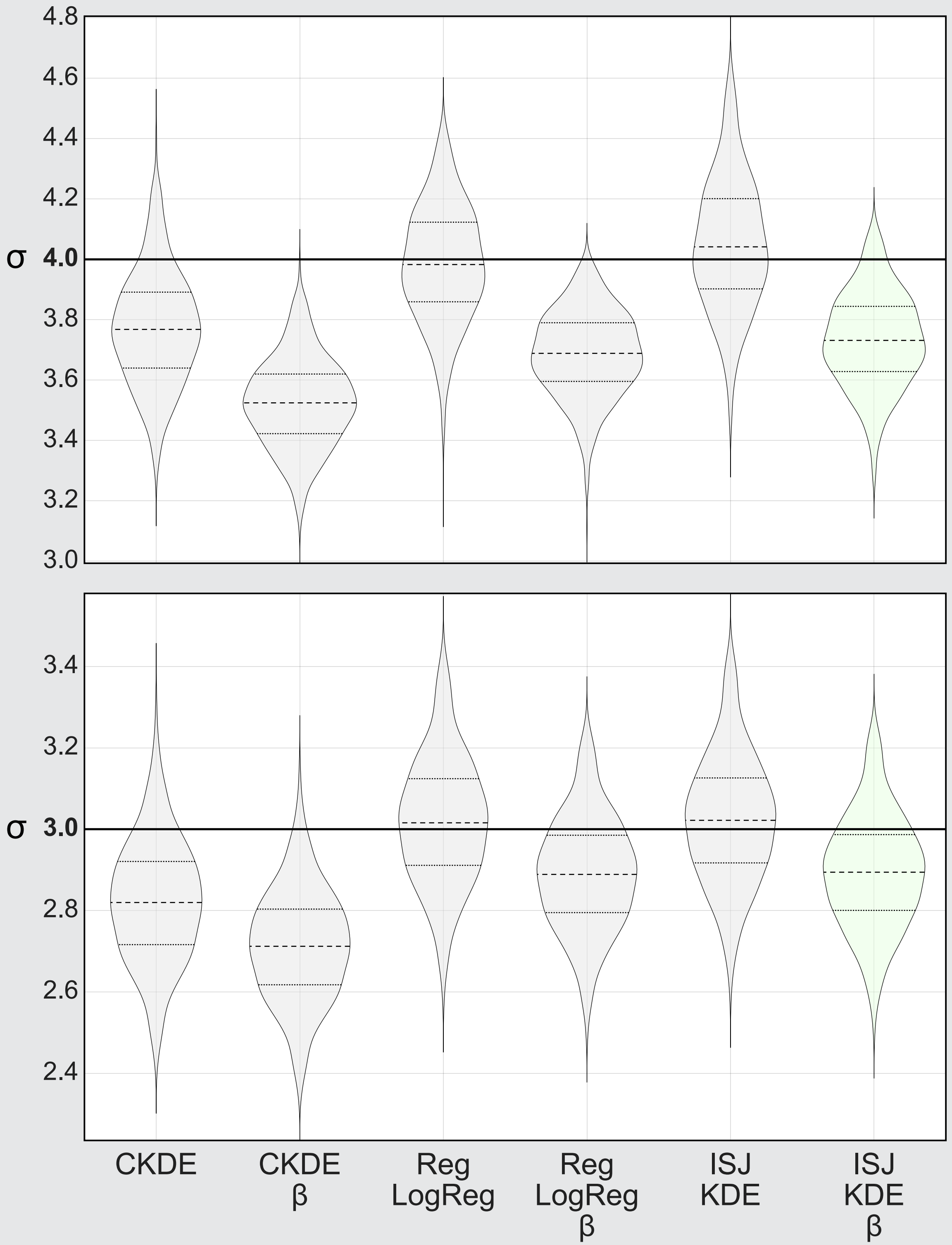

Bi-Gaussianized calibration

- 2026 paper
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  - σ ∈ {1,2,3,4,5,6}
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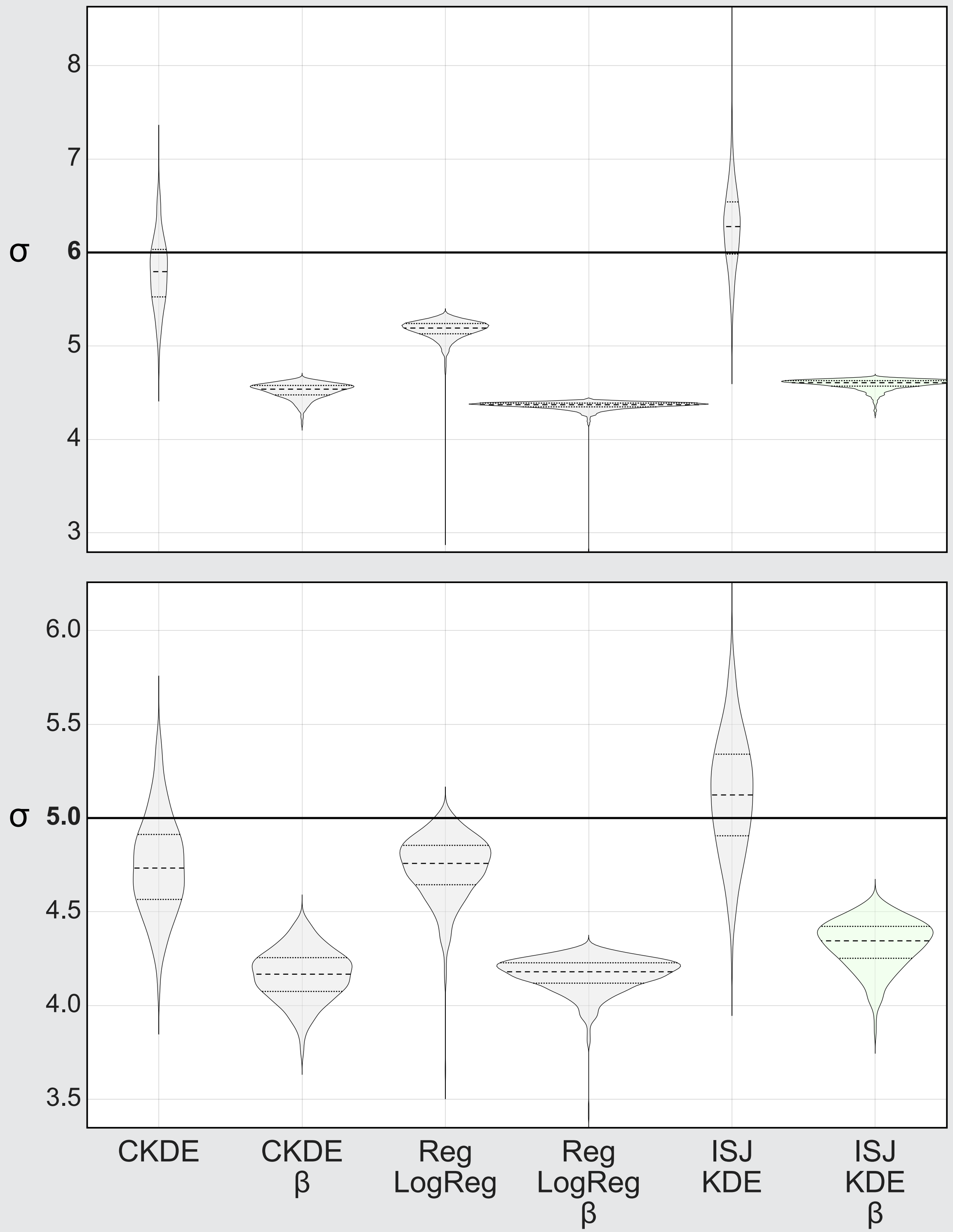
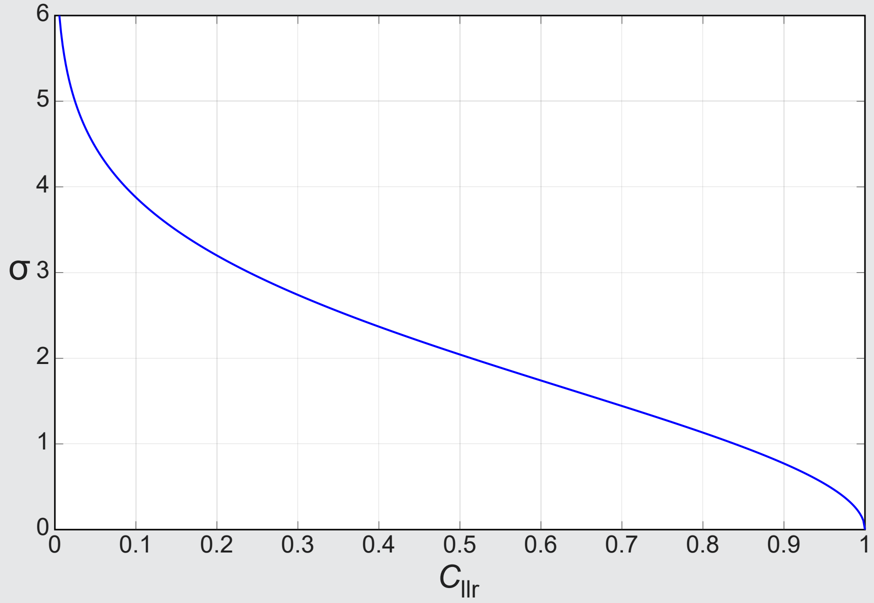


Bi-Gaussianized calibration

- ISJ-KDE-β method

- Method for selecting bandwidths for KDE models
    - Improved Sheather–Jones (ISJ) method (Botev et al., 2010)
    - does not use a Gaussian reference rule

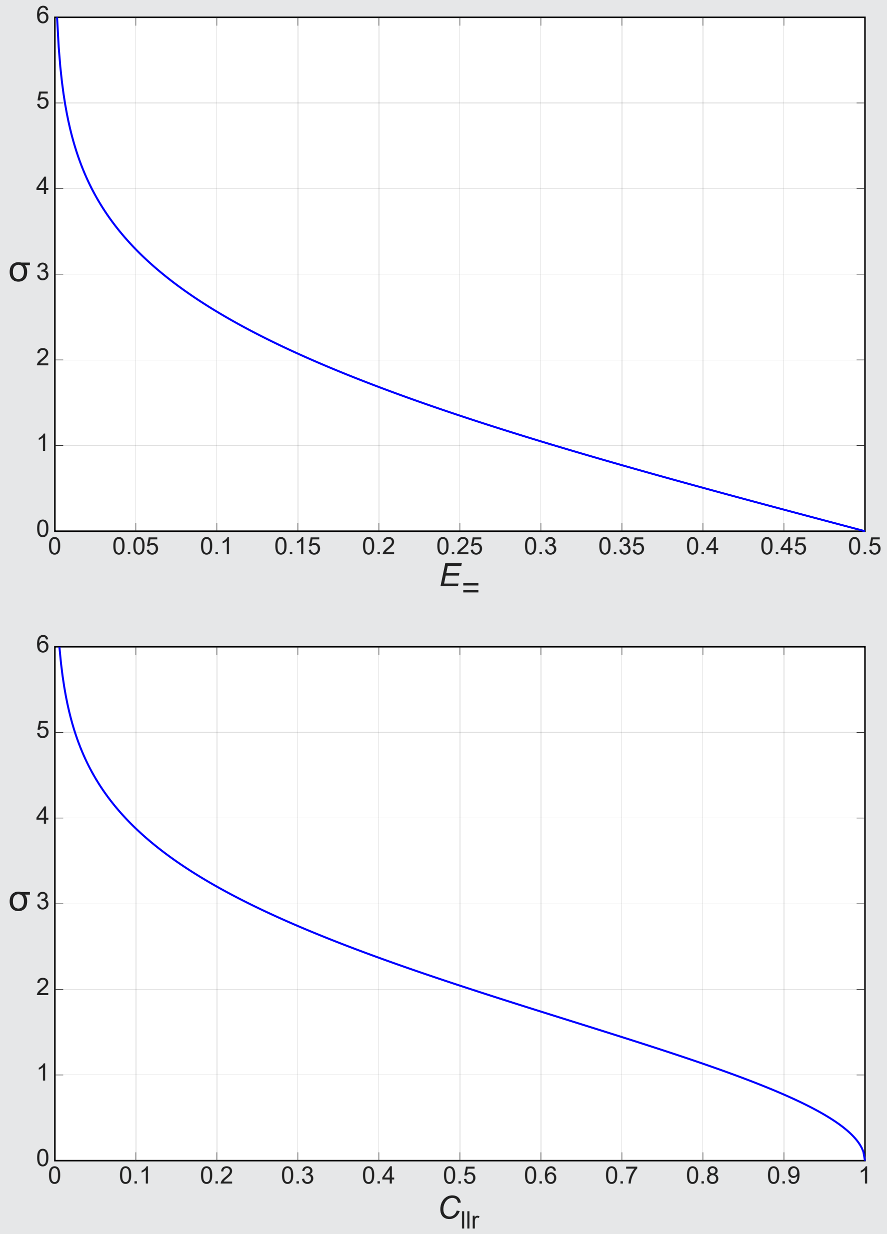
- C_{llr}-to-σ mapping
    - "exact" method
    - iterative search algorithm
    - does not use a fitted-curve approximation



Bi-Gaussianized calibration

- ISJ-KDE-β method

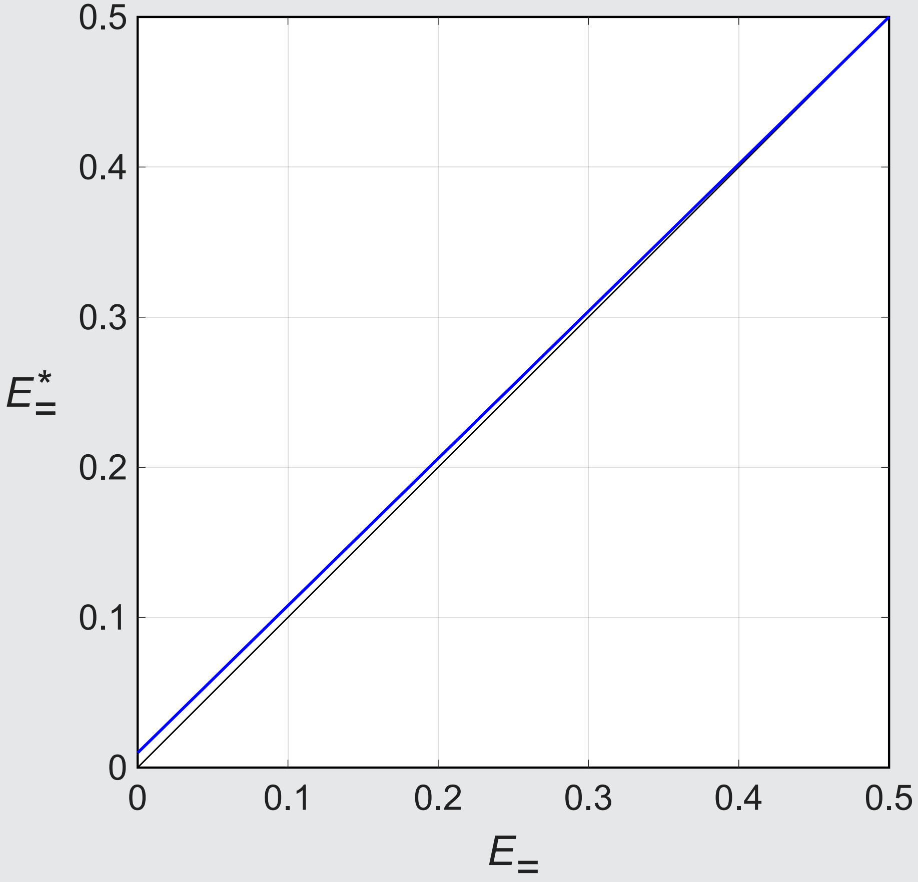
- Recalculate $E_=$ using a beta-binomial model with uniform priors
  - $C_{\mathrm{llr}} \longrightarrow \sigma$
  - $\sigma \longrightarrow E_=$
  - $E_=^* = \dfrac{n_s E_= + 1}{n_s + 2}$
  - $E_=^* \longrightarrow \sigma^*$

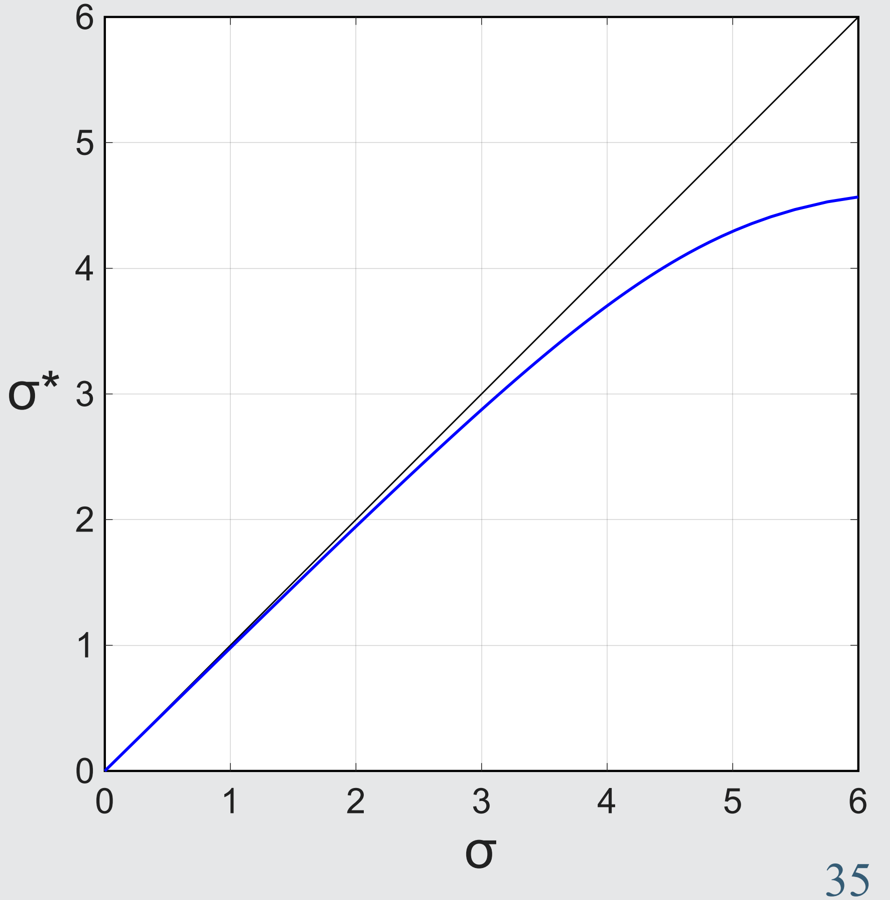


Bi-Gaussianized calibration

- ISJ-KDE-β method

- Recalculate $E_=$ using a beta-binomial model with uniform priors
    - $C_{\mathrm{llr}} \longrightarrow \sigma$
    - $\sigma \longrightarrow E_=$
    - $E_=^* = \dfrac{n_{\mathrm{s}} E_= + 1}{n_{\mathrm{s}} + 2}$
    - $E_=^* \longrightarrow \sigma^*$
    - $n_{\mathrm{s}} = 100$





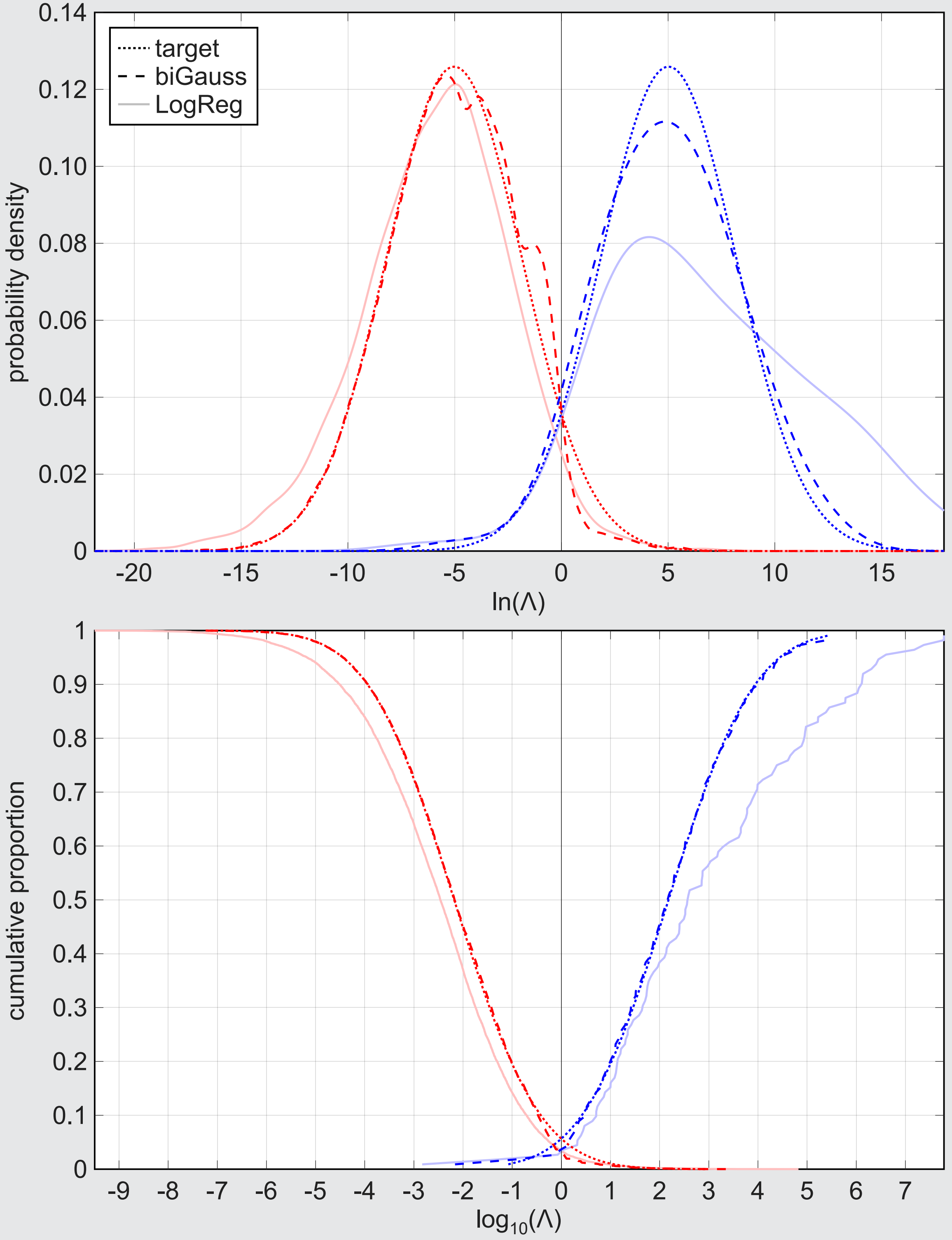
Bi-Gaussianized calibration

- ISJ-KDE-β method
- Recalculation of $E_=$ using a beta-binomial model with uniform priors
  - $E_=^* = \dfrac{n_s E_= + 1}{n_s + 2}$
  - $n_s = 100$

$\boldsymbol{\sigma}$	$\boldsymbol{E_=}$ (%)	$\boldsymbol{E_=^*}$ (%)	$\boldsymbol{\sigma^*}$
0	50	50	0
1	30.9	31.2	0.98
2	15.9	16.5	1.95
3	6.7	7.5	2.87
4	2.3	3.2	3.70
5	0.6	1.6	4.29
6	0.1	1.1	4.57
∞	0	1.0	4.67

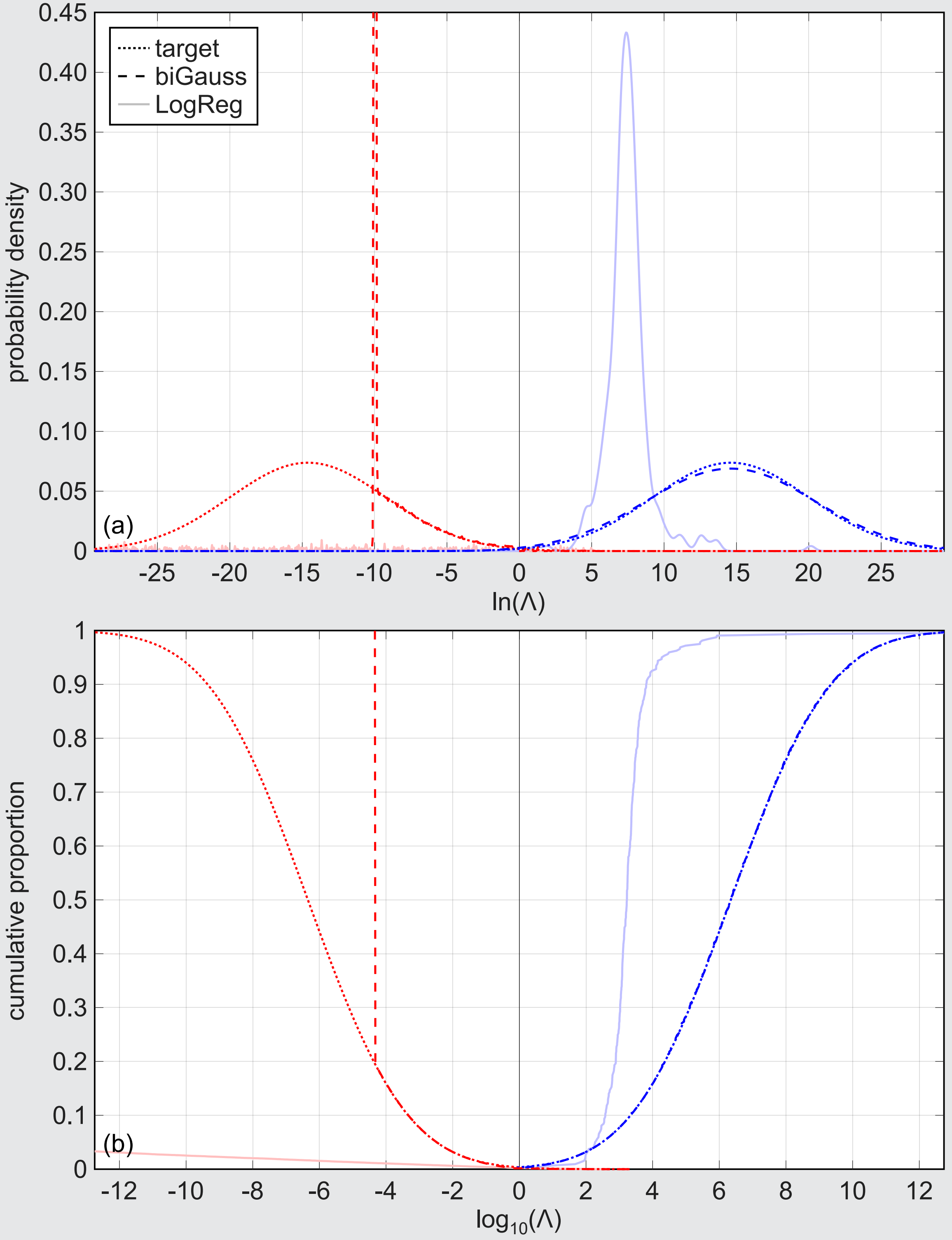
Bi-Gaussianized calibration

- Forensic-voice-comparison system
- ISJ-KDE-β method
- $n_s = 61$
- target $\sigma = 3.17$
- $C_{llr} = 0.176$



Bi-Gaussianized calibration

- Glass fragments
- ISJ-KDE-β method
- $n_s = 320$
- target $\sigma = 5.41$
- $C_{llr} = 0.007$



Conclusion

- If likelihood-ratio values are not calibrated, they are arbitrary numbers.

- For calibrated systems, the likelihood ratio of the likelihood ratio is the likelihood ratio.

- Bi-Gaussianized calibration monotonically maps uncalibrated log likelihood ratios toward the likelihood ratios of a perfectly calibrated bi-Gaussian system.

- Bi-Gaussianized calibration ISJ-KDE-β method
    - closer to perfect calibration that using traditional methods
    - reduces probability of overstating strength of evidence

Thank You

$$\Lambda = \frac{f(\Lambda \mid H_s)}{f(\Lambda \mid H_d)}$$